Introduction to the theory of voting I

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POLITICS • ELECTION

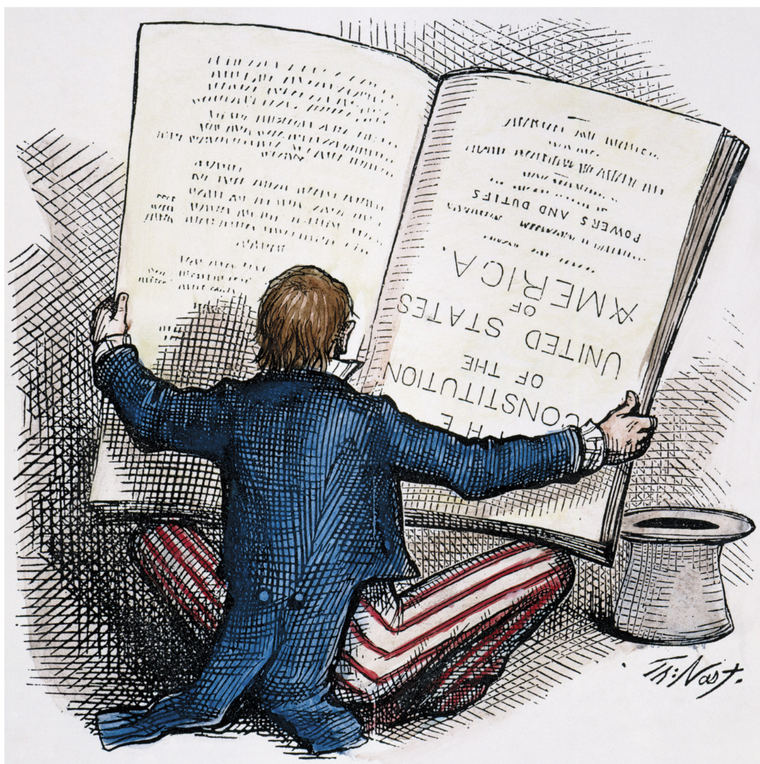
New York City Voters Just Adopted Ranked-Choice Voting in Elections. Here's How It Works

A poll workers explains the voting process to a voter at a public school polling location in New York on Nov. 5, 2019.
Gabriela Bhaskar—The New York Times/Redux

The Rules of the Game: A New Electoral System

Eric Maskin and Amartya Sen

JANUARY 19, 2017 ISSUE



Further Investigation

- EP, Voting Methods (Stanford Encyclopedia of Philosophy)
- C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
- M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)

Further Investigation

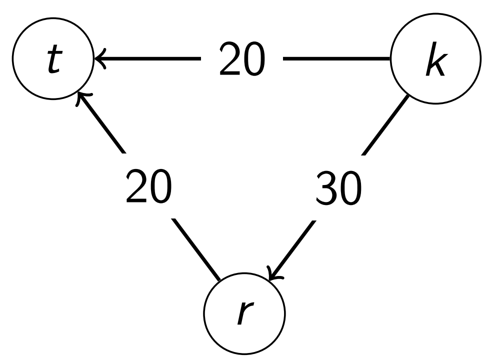
- `https://www.electology.org`
- `http://www.fairvote.org`
- `http://rangevoting.org`
- `https://www.opavote.com`
- `http://www.preflib.org`

Example

40	35	25
t	r	k
k	k	r
r	t	t

	t	k	r
t	0	40	40
k	60	0	65
r	60	35	0

Plurality winner	t
Runoff winner	r
Condorcet winner	k
Borda winner	k
Condorcet loser	t



Rankings

Let C be a set of candidates and V a set of voters.

A voter's *ranking* of the set of candidates is a strict linear order P on C: a relation $P \subseteq C \times C$ satisfying the following conditions for all $x, y, z \in C$:

asymmetry: if $x\ P\ y$ then *not* $y\ P\ x$;

transitivity: if $x\ P\ y$ and $y\ P\ z$, then $x\ P\ z$;

weak completeness: if $x \neq y$, then $x\ P\ y$ or $y\ P\ x$.

Let $L(C)$ be the set of all strict linear orders on C.

Profiles

A *profile* $\mathbf{P}$ *for* (C, V) is an element of $L(C)^V$, i.e., a function assigning to each $i \in V$ a relation $\mathbf{P}_i \in L(C)$.

- For $x, y \in C$, let $\mathbf{P}(x, y) = \{i \in V \mid x\mathbf{P}_i y\}$.
- For $x, y \in C$, let $\mathit{Margin}_{\mathbf{P}}(x, y) = |\mathbf{P}(x, y)| - |\mathbf{P}(y, x)|$

If $|C| = n$ and $|V| = m$, we call a profile for (C, V) an (n, m)*-profile*.

Voting Method

A *voting method* for (C, V) is a function assigning a nonempty subset of candidates, called the *winning set*, to each profile, i.e.,

$$f : L(C)^V \to \wp(C) \setminus \varnothing$$

Positional scoring rules

Suppose $\langle s_1, s_2, \ldots, s_n \rangle$ is a vector of numbers, called a *scoring vector*, where for each $l = 1, \ldots, n-1$, $s_l \geq s_{l+1}$.

Suppose $P \in L(C)$. The *score of* $x \in C$ *given* P is $score(P, x) = s_r$ where r is the *rank* of x in P.

For each profile $\mathbf{P}$ and $x \in C$, let $score(\mathbf{P}, x) = \sum_{i=1}^{n} score(\mathbf{P}_i, x)$.

A voting method f is a positional scoring rule for a scoring vector $\vec{s}$ provided that for all $\mathbf{P} \in L(C)^V$, $f(\mathbf{P}) = \mathrm{argmax}_{x \in C} score(\mathbf{P}, x)$.

`Borda`: $\langle n-1, n-2, \ldots, 1, 0 \rangle$.

`Plurality`: $\langle 1, 0, \ldots, 0 \rangle$.

Iterative procedures: Hare (Ranked-Choice, STV, ...)

- If some alternative is ranked first by an absolute majority of voters, then it is declared the winner.
- Otherwise, the alternative ranked first be the fewest voters (the plurality loser) is eliminated.
- Votes for eliminated alternatives get transferred: delete the removed alternatives from the ballots and "shift" the rankings (e.g., if 1st place alternative is removed, then your 2nd place alternative becomes 1st).

How should you deal with ties? (e.g., multiple alternatives are plurality losers)

Iterative procedures

Variants:

- Plurality with runoff: remove all candidates except top two plurality score;
- Coombs: remove candidates with most last place votes;
- Baldwin: remove candidate with smallest Borda score;
- Nanson: remove candidates with below average Borda score

Majority ordering/Margin graph

We say that a *majority prefers x to y* in $\mathbf{P}$, denoted $x >^{M}_{\mathbf{P}} y$, when

$$Margin_{\mathbf{P}}(x, y) > Margin_{\mathbf{P}}(y, x).$$

The *margin graph of* $\mathbf{P}$, $\mathcal{M}(\mathbf{P})$, is the weighted directed graph whose set of vertices is C with an edge from a to b weighted by $Margin(x, y)$ when $Margin(x, y) > 0$.

Condorcet criteria

The *Condorcet winner* in a profile $\mathbf{P}$ is a candidate $x \in C$ that is the maximum of the majority ordering, i.e., for all $y \in C$, if $x \neq y$, then $x >^M_{\mathbf{P}} y$.

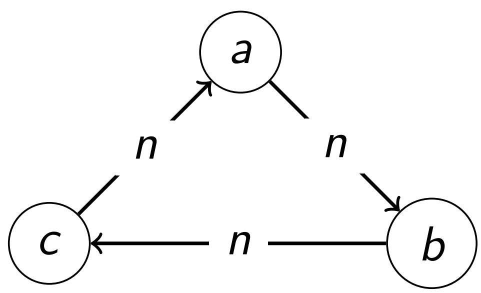
The *Condorcet loser* in a profile $\mathbf{P}$ is a candidate $x \in C$ that is the minimum of the majority ordering, i.e., for all $y \in C$, if $x \neq y$, then $y >^M_{\mathbf{P}} x$.

A voting method f is *Condorcet consistent*, if for all $\mathbf{P}$, if x is a Condorcet winner in $\mathbf{P}$, then $f(\mathbf{P}) = \{x\}$.

A voting method f is susceptible to the *Condorcet loser paradox* (also known as *Borda's paradox*) if there is some $\mathbf{P}$ such that x is a Condorcet loser in $\mathbf{P}$ and $x \in f(\mathbf{P})$.

Condorcet paradox

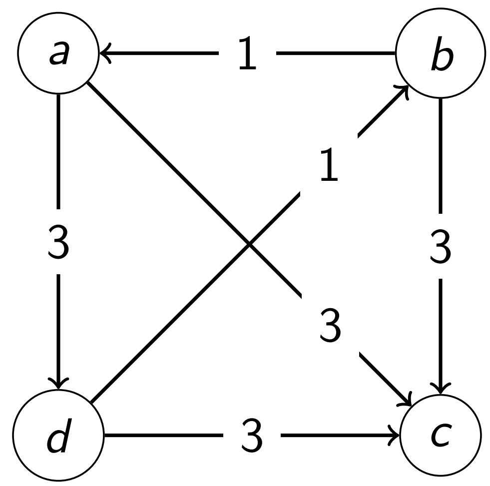
n	n	n
a	b	c
b	c	a
c	a	b



A voting method f is **resolute** if for all profiles $\mathbf{P}$, $|f(\mathbf{P})| = 1$.

Proposition (Moulin, 1983) Suppose that $m \geq 2$ is the number of alternatives and n is the number of voters. If n is divisible by any integer r with $1 < r < m$, then no neutral, anonymous, and Pareto voting method is resolute.

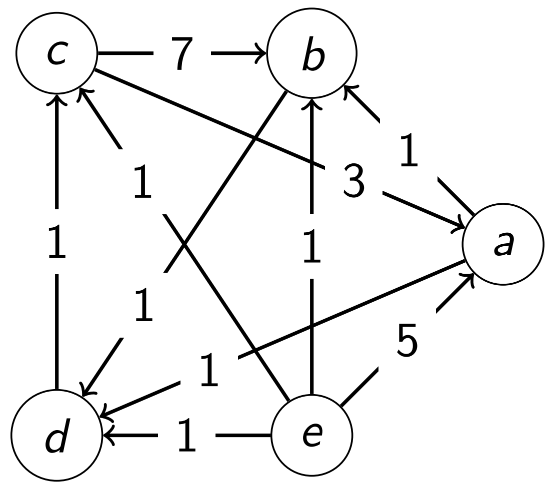
1	1	1	1	1	1	1	1	1
b	c	a	d	d	d	b	b	a
c	a	d	a	b	c	a	a	c
a	d	b	b	a	b	d	d	b
d	b	c	c	c	a	c	c	d



`Borda` winners	$\{a\}$
`Plurality` winners	$\{b, d\}$
`Runoff` winners	$\{d\}$
`Hare` winners	$\{a, b, d\}$
`Coombs` winners	$\{b\}$

- There is no Condorcet winner.
- c is the Condorcet loser.
- There is a top cycle.

1	1	1	1	1	1	1	1	2	1
b	e	c	e	c	c	e	d	d	a
d	a	b	c	b	e	a	c	e	c
e	b	e	a	a	d	d	a	c	b
a	d	a	b	d	a	c	b	b	e
c	c	d	d	e	b	b	e	a	d

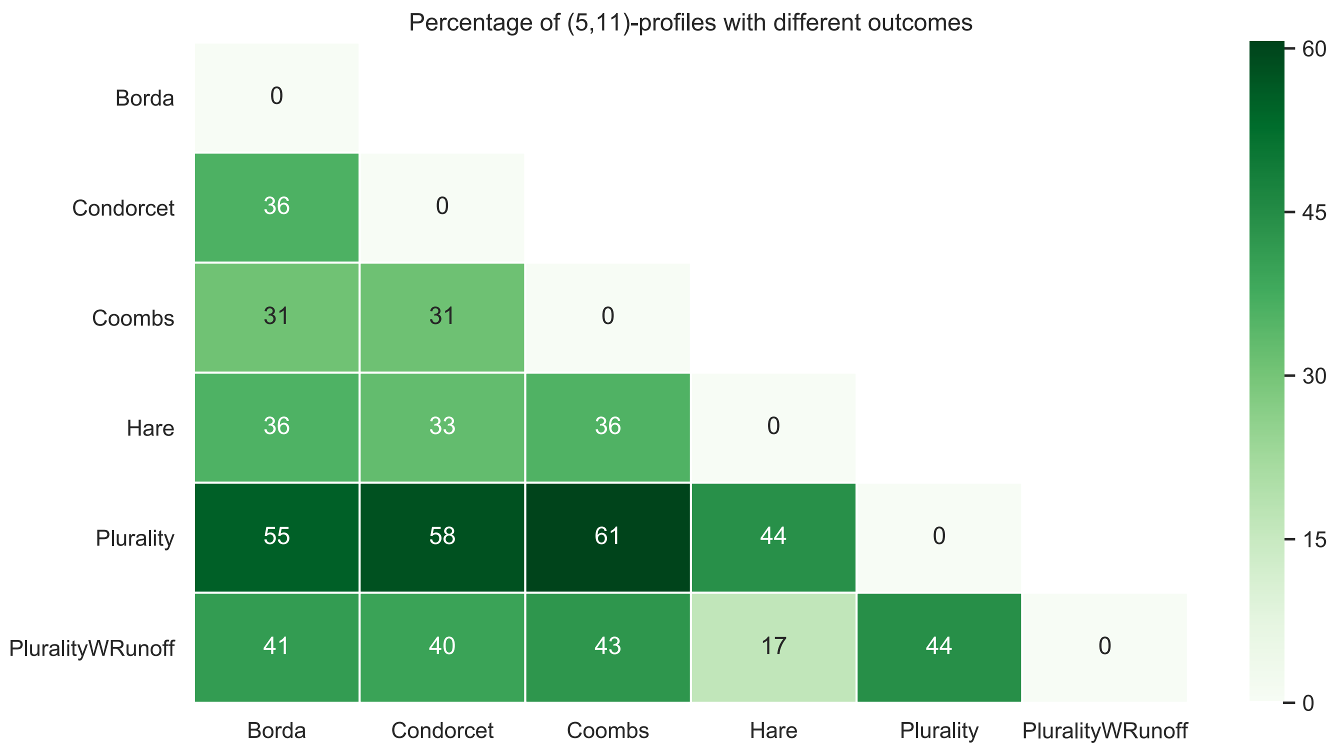


Borda winners	$\{c, e\}$
Plurality winners	$\{c, d, e\}$
Runoff winners	$\{c, d\}$
Hare winners	$\{d\}$
Coombs winners	$\{c\}$

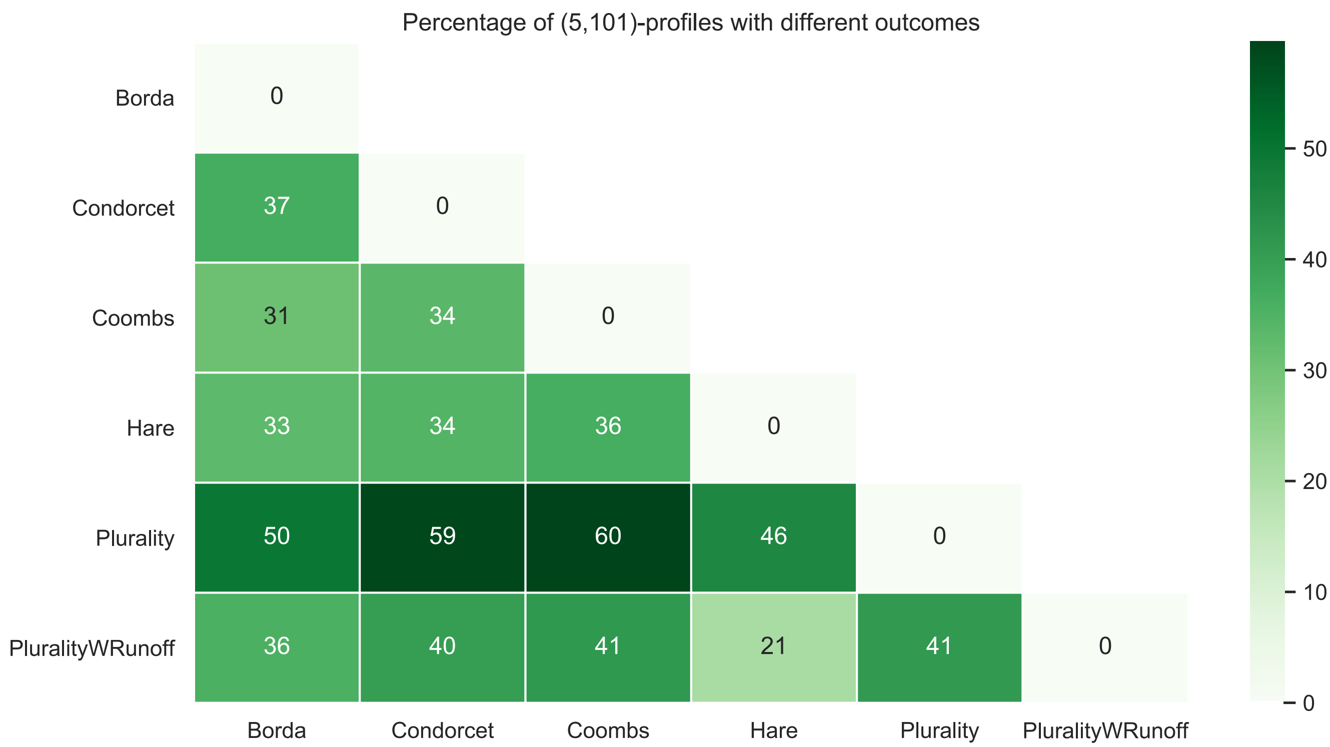
- The Condorcet winner is e.
- There is no Condorcet loser.

- Do the voting methods lead to different outcomes in practice?
- Should we always elect the Condorcet winner (if one exists)?

Different Voting Methods



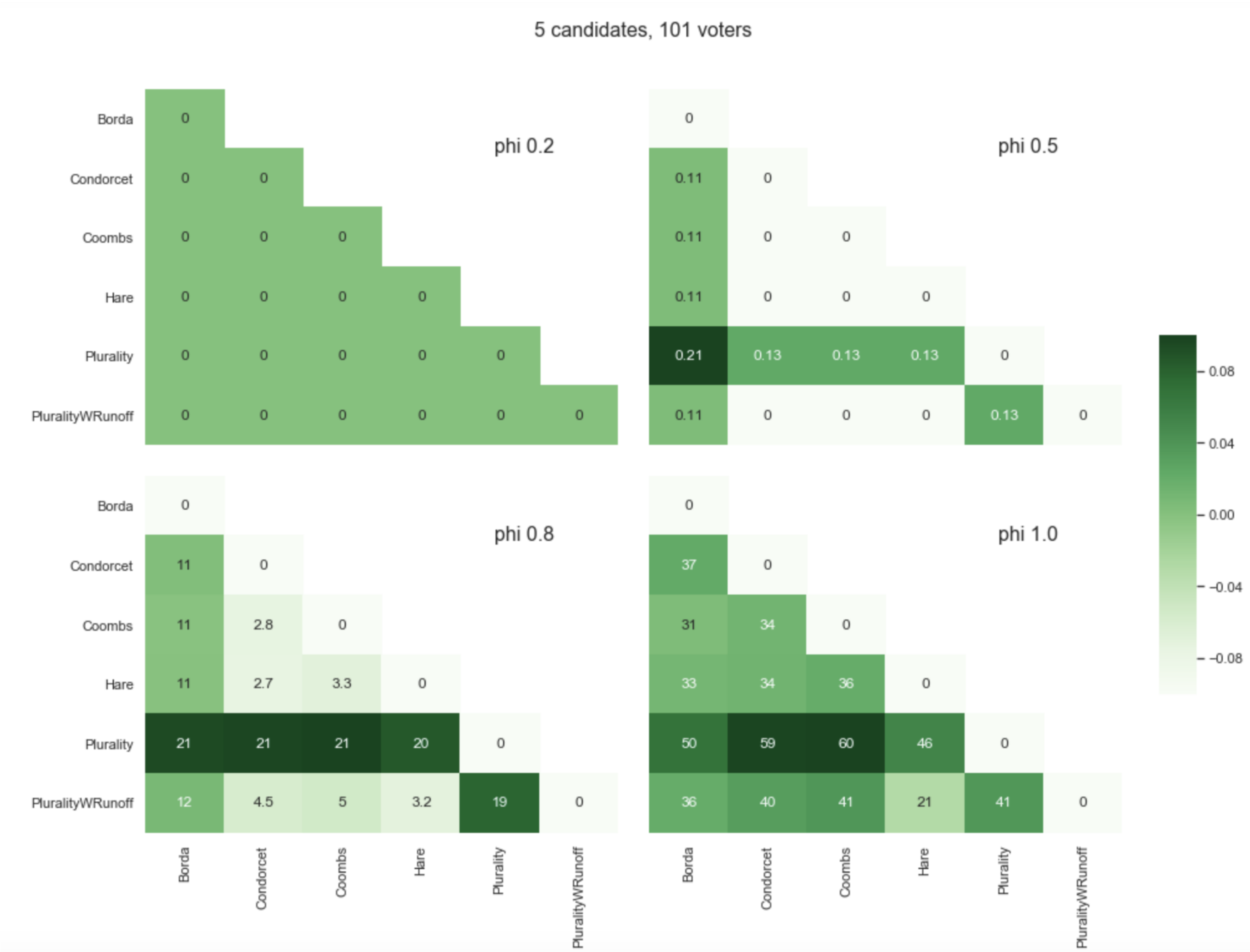
Different Voting Methods



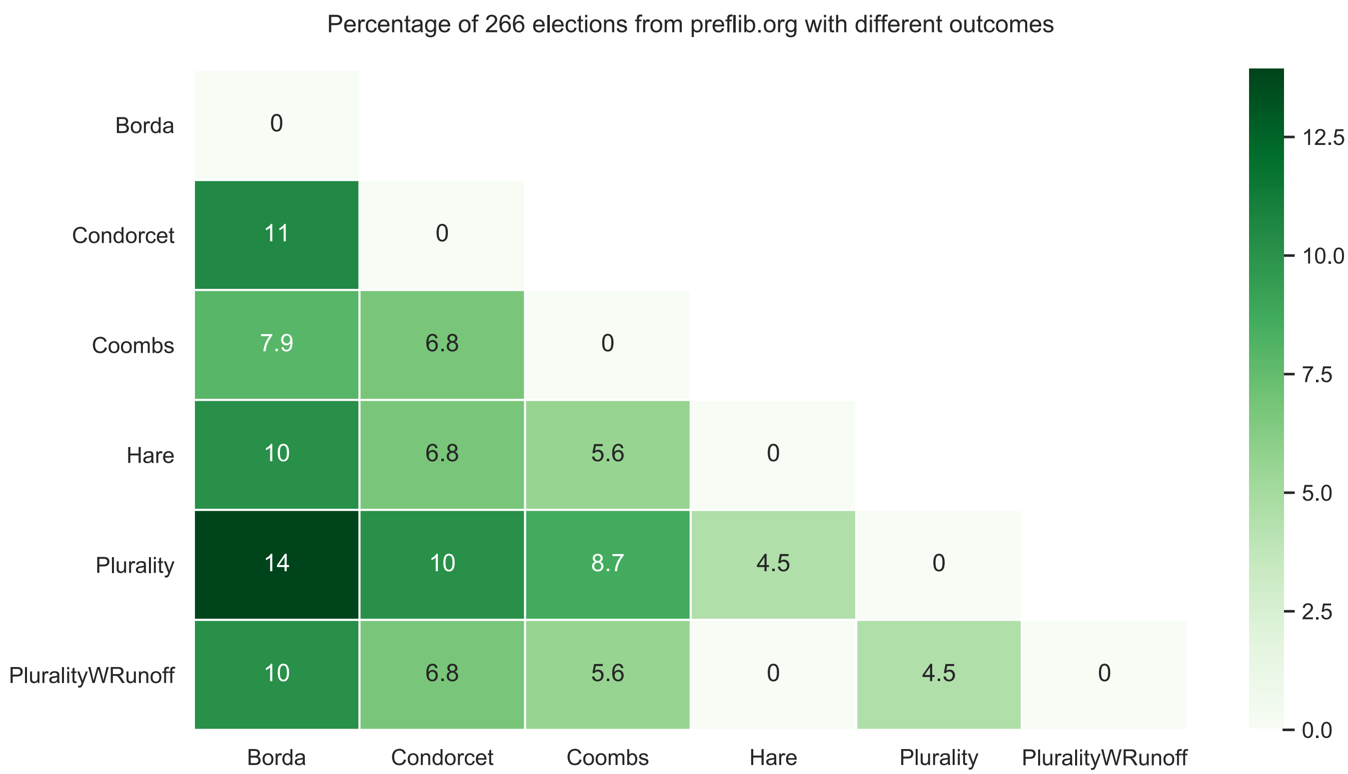
Models of voters behavior: IC (Impartial culture), IAC (Impartial anonymous culture), IANC (Impartial anonymous and neutral culture), Mallows models, Spatial models.

`http://preflib.org`

Different Voting Methods - Mallows Model



Different Voting Methods - Real Elections



F. Plassmann and T. N. Tideman. *How frequently do different voting rules encounter voting paradoxes in three-candidate elections?*. Social Choice and Welfare 42:31 - 75, 2014.

A. Popova, M. Regenwetter, and N. Mattei. *A Behavioral Perspective on Social Choice*. Annals of Mathematics and Artificial Intelligence, Volume 68, Number 1-3, 2013.

Should we always elect the Condorcet winner (if one exists)?

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
	a	a	b	b	c	c
	b	c	a	c	a	b
	c	b	c	a	b	a

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
2	a	a	b	b	c	c
1	b	c	a	c	a	b
0	c	b	c	a	b	a

$BS(a) = 2 \times 31 + 1 \times 39 + 0 \times 11 = 101$
$BS(b) = 2 \times 39 + 1 \times 31 + 0 \times 11 = 109$
$BS(c) = 2 \times 11 + 1 \times 11 + 0 \times 59 = 33$

$b >_{BC} a >_{BC} c$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
	a	a	b	b	c	c
	b	c	a	c	a	b
	c	b	c	a	b	a

$b >_{BC} a >_{BC} c$ $\quad$ $a >^{M} b >^{M} c$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
	a	a	b	b	c	c
	b	c	a	c	a	b
	c	b	c	a	b	a

$b >_{BC} a >_{BC} c$ $a >^{M} b >^{M} c$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
	a	a	b	b	c	c
	b	c	a	c	a	b
	c	b	c	a	b	a

$b >_{BC} a >_{BC} c$ $\qquad$ $a >^{M} b >^{M} c$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
s_2	a	a	b	b	c	c
s_1	b	c	a	c	a	b
s_0	c	b	c	a	b	a

Condorcet's Other Paradox: No *scoring rule* will work...

$b >_{BC} a >_{BC} c$ $\qquad$ $a >^{M} b >^{M} c$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
s_2	a	a	b	b	c	c
s_1	b	c	a	c	a	b
s_0	c	b	c	a	b	a

Condorcet's Other Paradox: No *scoring rule* will work...
$Score(a) = s_2 \times 31 + s_1 \times 39 + s_0 \times 11$
$Score(b) = s_2 \times 39 + s_1 \times 31 + s_0 \times 11$

$b >_{BC} a >_{BC} c$ $\quad$ $a >^M b >^M c$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
s_2	a	a	b	b	c	c
s_1	b	c	a	c	a	b
s_0	c	b	c	a	b	a

Condorcet's Other Paradox: No *scoring rule* will work...

$Score(a) = s_2 \times 31 + s_1 \times 39 + s_0 \times 11$

$Score(b) = s_2 \times 39 + s_1 \times 31 + s_0 \times 11$

$Score(a) > Score(b) \Rightarrow 31s_2 + 39s_1 > 39s_2 + 31s_1 \Rightarrow s_1 > s_2$

$b >_{BC} a >_{BC} c$ $\qquad$ $a >^M b >^M c$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
s_2	a	a	b	b	c	c
s_1	b	c	a	c	a	b
s_0	c	b	c	a	b	a

Theorem (Fishburn 1974). For all $m \geq 3$, there is some voting situation with a Condorcet winner such that every scoring rule will have at least $m-2$ candidates with a greater score than the Condorcet winner.

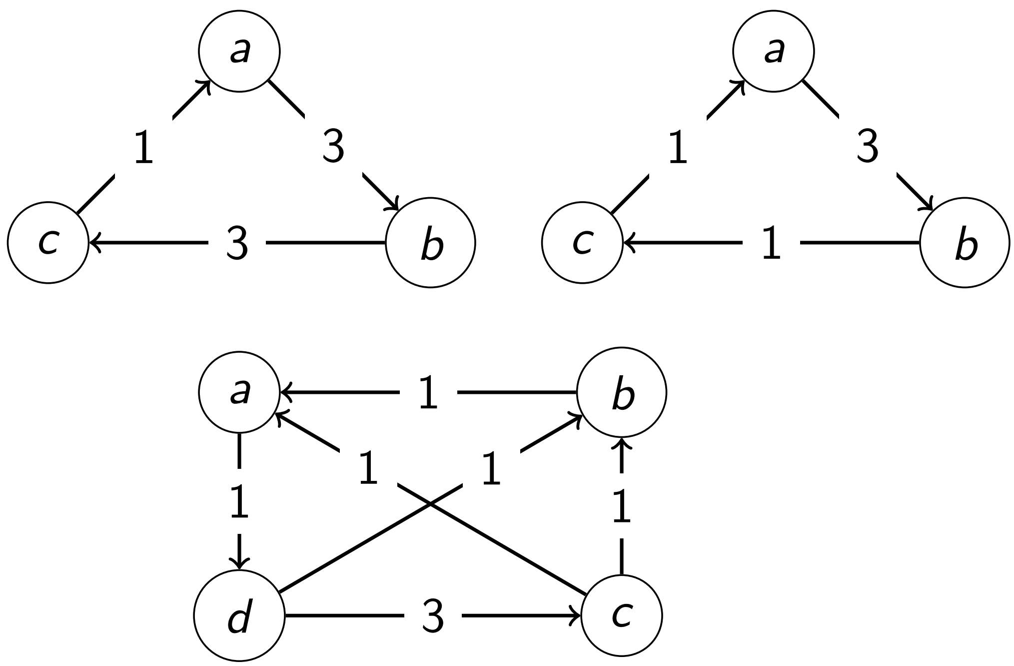
P. Fishburn. *Paradoxes of Voting*. The American Political Science Review, 68:2, pgs. 537 - 546, 1974.

Saari's argument, Balinski and Laraki (2010, pg. 77); Zwicker (2016, Proposition 2.5): Multiple districts paradox, f *cancels properly*.

2	2	2	1	2
a	b	c	a	b
b	c	a	b	a
c	a	b	c	c

- no Condorcet winner in the left profile
- b is the Condorcet winner in the right profile
- a is the Condorcet winner in the combined profiles

Not All Cycles are Created Equal



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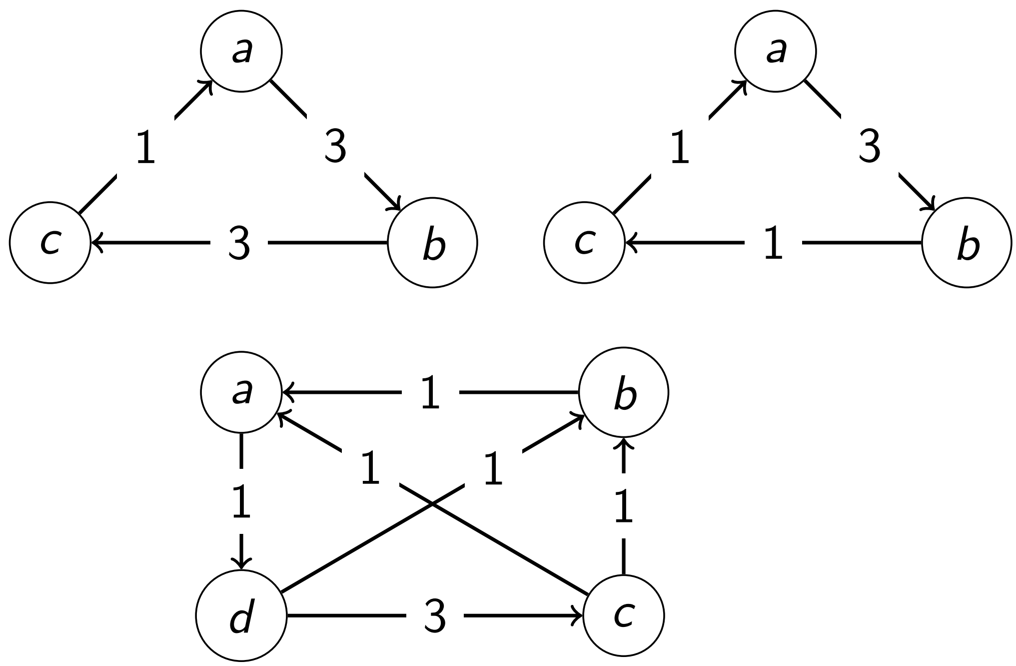
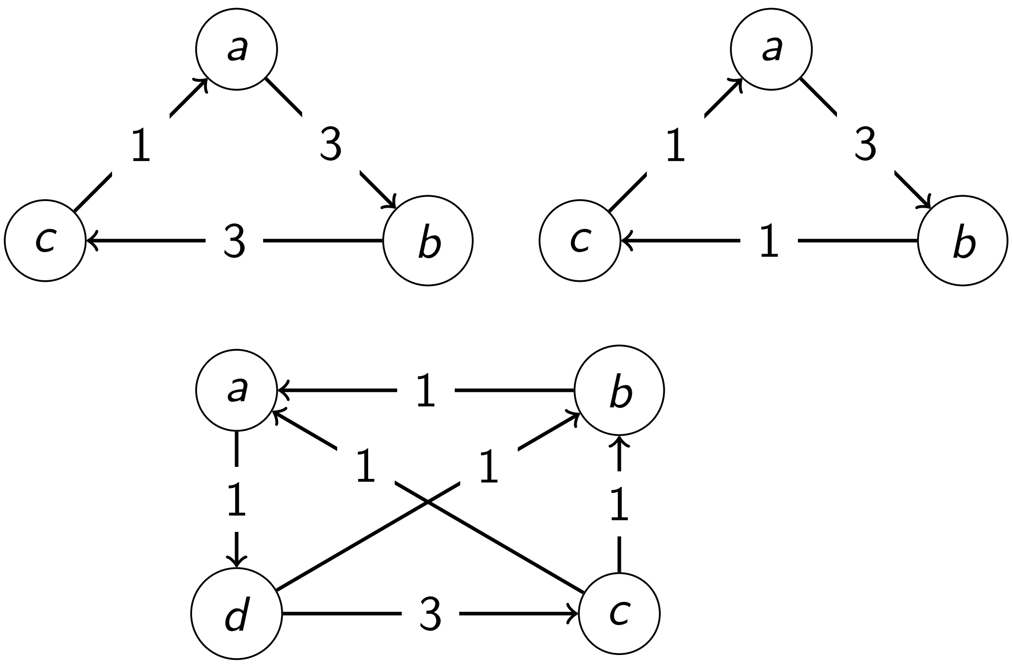


MiniMax: pick the candidates whose worst defeat is the smallest.
Copeland: pick the candidates with the best win-loss record.

Not All Cycles are Created Equal



MiniMax: pick the candidates whose worst defeat is the smallest.
Copeland: pick the candidates with the best win-loss record.

Can we do better?

Aside: McGarvey's Theorem

Theorem (McGarvey 1953)

If G is any directed graph with $k \geq 2$ nodes, there exists a profile of $4k$ voters such that there is an edge from x to y when $x >^{M}_{\mathbf{P}} y$.

D.C. McGarvey. *A Theorem on the Construction of Voting Paradoxes*. Econometrica, 21, pgs. 608 - 610, 1953.

Fishburn's Classification

Classify voting rules on the basis of the information they require.

- C1: Winners can be computed from the majority graph alone. Examples: Copeland
- C2: Winners can be computed from the weighted majority graph (but not from the majority graph alone). Examples: Minimax, Borda (think about it!)
- C3: All other voting rules. Examples: Ranked-Choice, Young, Dodgson

P.C. Fishburn. *Condorcet Social Choice Functions*. SIAM Journal on Applied Mathematics, 33(3):469 - 489, 1977.

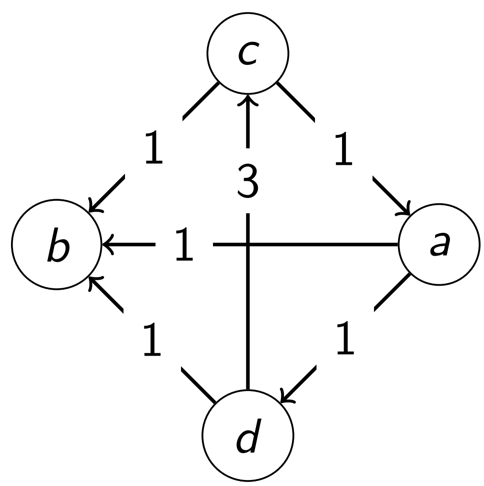
Young: Elect alternative x that minimises the number of voters we need to remove before x becomes the Condorcet winner.

Dodgson: Elect alternative x that minimises the number of swaps of adjacent alternatives in the profile we need to perform before x becomes the Condorcet winner.

Condorcet Loser Paradox

Consider the following profile **P** with 5 voters and 4 alternatives:

1	1	1	1	1
a	a	d	c	b
b	d	c	a	d
d	c	b	d	c
c	b	a	b	a



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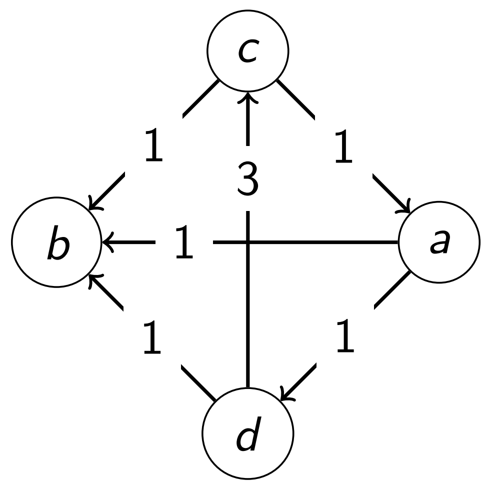

$MiniMax(\mathbf{P}) = \{a, b, d\}$, but b is the Condorcet loser.

Monotonicity

Definition. For any profiles $\mathbf{P}$ and $\mathbf{P}'$ with $V(\mathbf{P}) = V(\mathbf{P}')$ and $x \in X(\mathbf{P}) = X(\mathbf{P}')$, we say that $\mathbf{P}'$ *is obtained from* $\mathbf{P}$ *by a simple lift of* x if the following conditions hold:

1. for all $a, b \in X(\mathbf{P}) \setminus \{x\}$ and $i \in V$, $a\mathbf{P}_i b$ iff $a\mathbf{P}'_i b$;
2. for all $a \in X(\mathbf{P})$ and $i \in V$, if $x\mathbf{P}_i a$ then $x\mathbf{P}'_i a$.

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2. for all $a \in X(\mathbf{P})$ and $i \in V$, if $x\mathbf{P}_i a$ then $x\mathbf{P}'_i a$.

Definition. A voting method F satisfies *monotonicity* if for any profile $\mathbf{P}$ and $x \in X(\mathbf{P})$, if $x \in F(\mathbf{P})$ and $\mathbf{P}'$ is obtained from $\mathbf{P}$ by a simple lift of x, then $x \in F(\mathbf{P}')$ and $F(\mathbf{P}') \subseteq F(\mathbf{P})$.

Monotonicity

A candidate receiving more “support” shouldn’t make her worse off.

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More-is-Less Paradox: If a candidate C is elected under a given a profile of rankings of the competing candidates, it is possible that, *ceteris paribus*, C may not be elected if some voter(s) raise C in their rankings.

P. Fishburn and S. Brams. *Paradoxes of Preferential Voting*. Mathematics Magazine (1983).

More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	a	c	b	b
	b	a	c	a
	c	b	a	c

# voters	6	5	4	2
	a	c	b	a
	b	a	c	b
	c	b	a	c

More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	a	*c*	*b*	*b*
	b	*a*	*c*	*a*
	c	*b*	*a*	*c*

# voters	6	5	4	2
	a	*c*	*b*	*a*
	b	*a*	*c*	*b*
	c	*b*	*a*	*c*

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	c	*b*	*a*	*c*

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	a	*c*	*b*	*a*
	b	*a*	*c*	*b*
	c	*b*	*a*	*c*

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	a	*c*	*b*	*a*
	b	*a*	*c*	*b*
	c	*b*	*a*	*c*

More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	a	c	b	b
	b	a	c	a
	c	b	a	c

Winner: a

# voters	6	5	4	2
	a	c	b	a
	b	a	c	b
	c	b	a	c

More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	a	c	b	b
	b	a	c	a
	c	b	a	c

Winner: a

# voters	6	5	4	2
	a	c	b	a
	b	a	c	b
	c	b	a	c

More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	a	*c*	*b*	*b*
	b	*a*	*c*	*a*
	c	*b*	*a*	*c*

Winner: *a*

# voters	6	5	4	2
	a	*c*	*b*	*a*
	b	*a*	*c*	*b*
	c	*b*	*a*	*c*

Winner: *c*

More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	a	*c*	*b*	*b*
	b	*a*	*c*	*a*
	c	*b*	*a*	*c*

Winner: *a*

# voters	6	5	4	2
	a	*c*	*b*	*a*
	b	*a*	*c*	*b*
	c	*b*	*a*	*c*

Winner: *c*

Monotonicity: A candidate receiving more “support” shouldn't make her worse off.

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No-Show Paradox: A voter may obtain a more preferable outcome if he decides not to participate in an election than, *ceteris paribus*, if he decides to participate in the election.

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- **Twin Paradox**: A voter may obtain a less preferable outcome if his “twin” (a voter with the exact same ranking) decides to participate in the election.

Monotonicity: A candidate receiving more “support” shouldn’t make her worse off

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- **Twin Paradox**: A voter may obtain a less preferable outcome if his “twin” (a voter with the exact same ranking) decides to participate in the election.
- **Truncation Paradox**: A voter may obtain a more preferable outcome if, *ceteris paribus*, he only reveals part of his ranking of the candidates.

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	a	b	c	c
	b	c	a	b
	c	a	b	a

# voters	2	3	1	3
	a	b	c	c
	b	c	a	b
	c	a	b	a

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

# voters	2	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

Winner: *c*

# voters	2	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

Winner: *c*

# voters	2	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	a	b	c	c
	b	c	a	b
	c	a	b	a

Winner: c

# voters	2	3	1	3
	a	b	c	c
	b	c	a	b
	c	a	b	a

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

Winner: *c*

# voters	2	3	1	3
	a	*b*	*c*	*c*
	b	*c*	*a*	*b*
	c	*a*	*b*	*a*

Winner: *b*

Twin Paradox: Plurality with Runoff

# voters	4	3	1	3
	a	b	c	c
	b	c	a	b
	c	a	b	a

Winner: c

# voters	2	3	1	3
	a	b	c	c
	b	c	a	b
	c	a	b	a

Winner: b

Failures of Monotonicity

Theorem (Smith 1973) No point runoff system involving two or more stages and non-trivial point systems is monotonic. More precisely, if such a system determines first place first, then a change of votes in a candidate's favor can remove him from first place. If it determines last place first, such a change can put a candidate in last place who was not previously there.

J. H. Smith. *Aggregation of Preferences with Variable Electorate*. Econometrica, 41(6), pp. 1027-1041, 1973.

D. Felsenthal and N. Tideman. *Varieties of Failure of Monotonicity and Participation under Five Voting Methods*. Theory and Decision, 75, pgs. 59 - 77, 2013.

Failures of Monotonicity

Example: Burlington, VT 2009 Mayoral Race
(rangevoting.org/Burlington.html)

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Theorem (Moulin). If there are four or more candidates, then every Condorcet consistent voting methods is susceptible to the No-Show paradox.

H. Moulin. *Condorcet's Principle Implies the No Show Paradox*. Journal of Economic Theory, 45, pgs. 53 - 64, 1988.

For a profile $\mathbf{P}$, $X(\mathbf{P})$ are the candidates in $\mathbf{P}$ and $V(\mathbf{P})$ are the voters in $\mathbf{P}$

Independence of Clones

d and p are "clones" of each other in the sense that they appear next to each other on every ballot:

37	29	34
r	d	p
d	p	d
p	r	r

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Definition

Given a profile $\mathbf{P}$, $C \subseteq X(\mathbf{P})$ is a *set of clones for* $\mathbf{P}$ iff for every $i \in V$, $x, y \in C$, and $z \in X(\mathbf{P}) \setminus C$, either $x\mathbf{P}_i z$ and $y\mathbf{P}_i z$, or $z\mathbf{P}_i x$ and $z\mathbf{P}_i y$.

Independence of Clones

For $x \in X(\mathbf{P})$, let $\mathbf{P}_{-x}$ be the profile resulting from removing x from the ballots of $\mathbf{P}$.

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Definition

A voting method F is such that *non-clone choice is independent of clones* if for all profiles $\mathbf{P}$, sets C of clones of $\mathbf{P}$, $c \in C$, and $a \in X(\mathbf{P}) \setminus C$, we have $a \in F(\mathbf{P})$ iff $a \in F(\mathbf{P}_{-c})$.

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The mayoral election shows that Plurality violates this axiom:

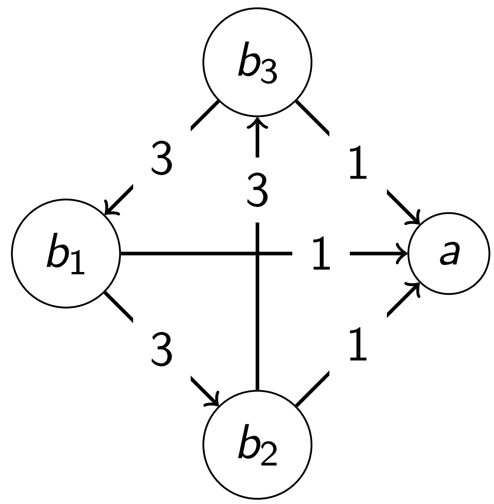
37	29	34
r	d	p
d	p	d
p	r	r

37	29	34
r	d	d
d	r	r

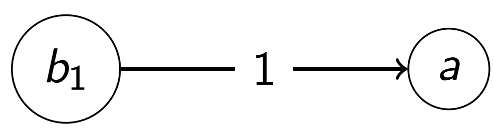
Independence of Clones

The following example shows that MiniMax violates the axiom:

3	1	3	2
a	a	b_2	b_3
b_1	b_3	b_3	b_1
b_2	b_1	b_1	b_2
b_3	b_2	a	a



4	5
a	b_1
b_1	a



Independence of Clones

Definition

F is such that *clone choice is independent of clones* if for all profiles $\mathbf{P}$, sets C of clones of $\mathbf{P}$, and $c \in C$, we have

$$C \cap F(\mathbf{P}) \neq \varnothing \text{ iff } C \setminus \{c\} \cap F(\mathbf{P}_{-c}) \neq \varnothing.$$

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Finally, F satisfies *independence of clones* if F is such that non-clone choice is independent of clones and clone choice is independent of clones.

Methods Left Standing

Two Condorcet consistent, monotonic, clone-independent methods:

- Ranked Pairs: Order the edges by their weights, “lock” in an edge one at time (unless it creates a cycles)
- Beat Path: *a beats b* when the minimum weight of a path from *a* to *b* is greater than the minimum weight on a path from *b* to *a*

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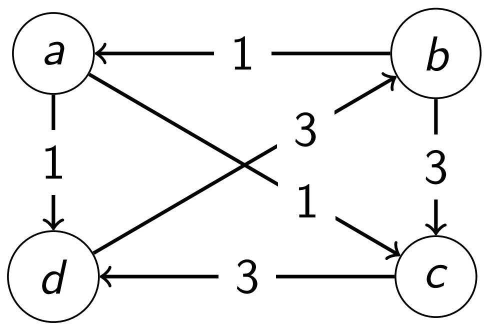
An even better one! SplitCycle (current work with Wes Holliday)

T. M. Zavist and T. N. Tideman. *Complete independence of clones in the ranked pairs rule*. Social Choice and Welfare 6(2):167 - 173, 1989.

M. Brill and F. Fischer. *The Price of Neutrality for the Ranked Pairs Method*. Proceedings of the Twenty-Sixth AAAI Conference on Artificial Intelligence.

W. Holliday and EP. *Split Cycle: A New Condorcet Consistent Voting Method Independent of Clones and Immune to Spoilers*. manuscript.



Copeland:	$\{a, b\}$
Minimax:	$\{a\}$
Ranked Pairs:	$\{b, c, d\}$
Beatpath:	$\{a, b, c, d\}$

- Impossibility theorems
- Probabilistic social choice
- Characterization results/Voting methods as statistical estimators
- Strategic voting
- Behavioral social choice