

# APPLIED MECHANISM DESIGN FOR SOCIAL GOOD

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Lecture #7 – 02/18/2020

**CMSC828M**  
**Tuesdays & Thursdays**  
**2:00pm – 3:15pm**



**COMPUTER SCIENCE**  
UNIVERSITY OF MARYLAND

# ANNOUNCEMENTS

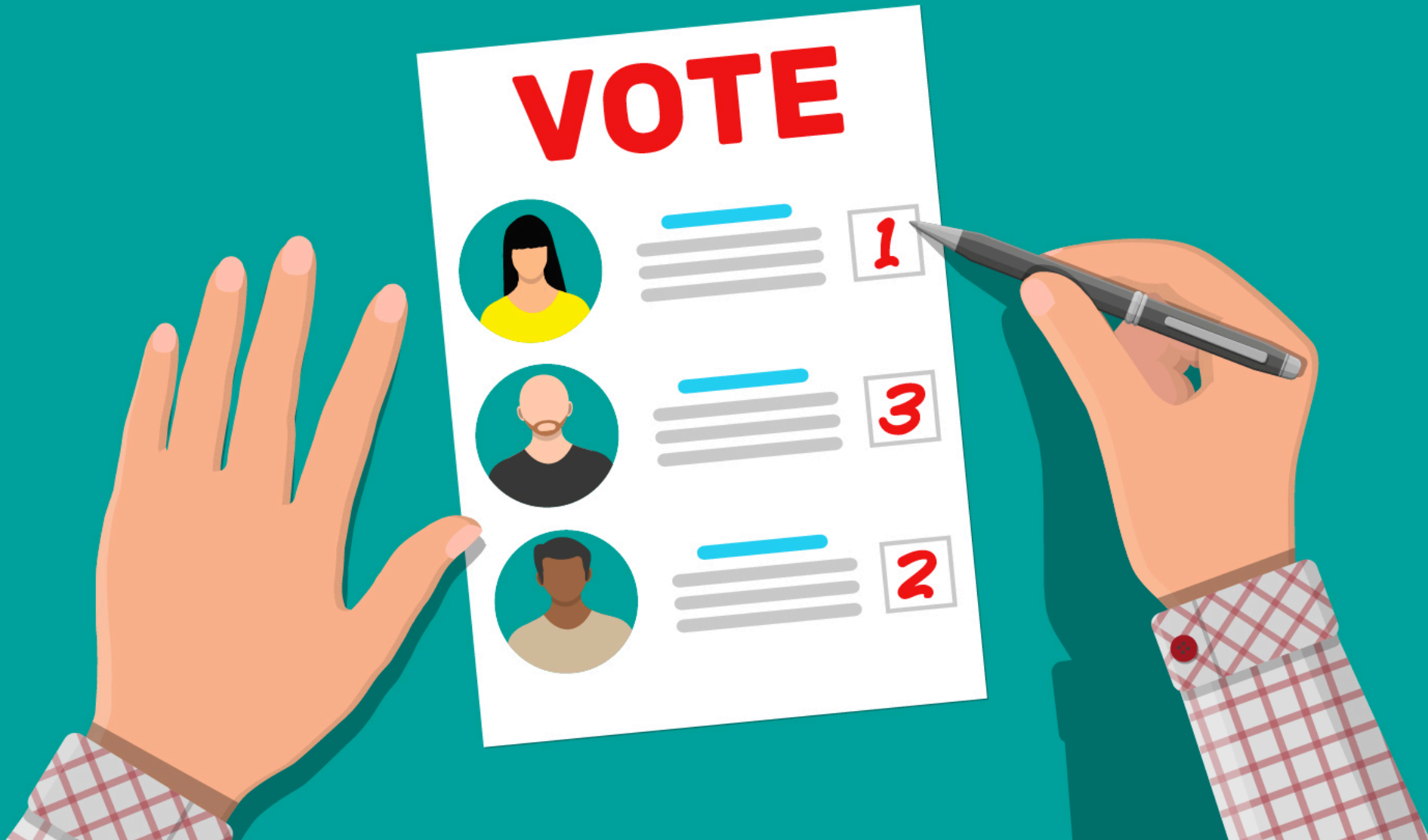
**Please do start thinking about your course projects!**

**Can be individual or group:**

- No hard limit on group size (but 1-3 seems to work best!)
- One place to find partners: [cmssc828m.slack.com](https://cmssc828m.slack.com)
- Another place to find partners: right here!

**Talk to me (Slack, office hours, etc), talk to Eric, talk to my PhD students, talk to each other, talk to other professors, ask the Internet, ask your friends in other industries, et cetera!**

# THANK YOU, ERIC!



*Unironically-chosen ranked choice voting image --^*

# COMPUTATIONAL SOCIAL CHOICE

There are many strong **impossibility results** like Gibbard–Satterthwaite & Arrow’s “Possibility” Theorem

- We may discuss more in the future, but also talk with Eric!

**Computational social choice** creates “well-designed” implementations of social choice functions, with an eye toward:

- Computational tractability of the winner determination problem
- Communication complexity of preference elicitation
- Designing the **mechanism** to elicit preferences **truthfully**

Interactions between these can lead to positive theoretical results and practical circumventions of impossibility results.

# MECHANISM DESIGN: MODEL

Before: we were **given** preference profiles

Reality: agents **reveal** their (private) preferences

- Won't be truthful unless it's in their **individual** interest; but
- We want some **globally** good outcome

**Formally:**

- Center's job is to pick from a set of outcomes  $O$
- Agent  $i$  draws a private type  $\theta_i$  from  $\Theta_i$ , a set of possible types
- Agent  $i$  has a public valuation function  $v_i : \Theta_i \times O \rightarrow \mathbb{R}$
- Center has public objective function  $g : \Theta \times O \rightarrow \mathbb{R}$ 
  - Social welfare max aka efficiency, maximize  $g = \sum_i v_i(\theta_i, o)$
  - Possibly plus/minus monetary payments

# MECHANISM DESIGN WITHOUT MONEY

A (direct) **deterministic mechanism without payments**  $\sigma$  maps  $\Theta \rightarrow O$

A (direct) **randomized mechanism without payments**  $\sigma$  maps  $\Theta \rightarrow \Delta(O)$ , the set of all probability distributions over  $O$

Any mechanism  $\sigma$  induces a Bayesian **game**,  $\text{Game}(\sigma)$ :

- Bayesian game: agents have incomplete information about other agents (e.g., may not know player types)

A mechanism is said to **implement** a social choice function  $f$  if, for every input (e.g., preference profile), there is a Nash equilibrium for  $\text{Game}(\sigma)$  where the outcome is the same as  $f$

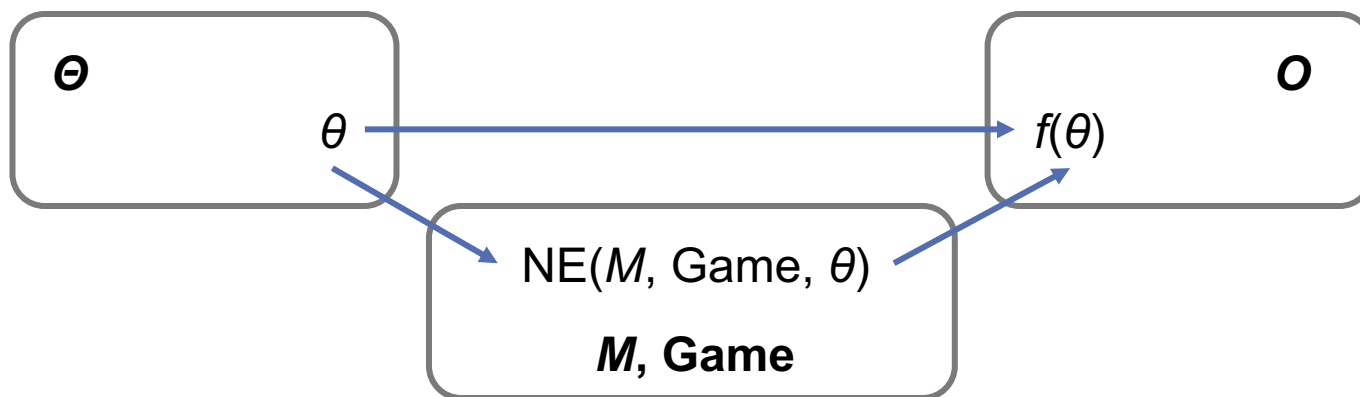
# PICTORIALLY ...

Agents draw private types  $\theta$  from  $\Theta$

If those types were known, an outcome  $f(\theta)$  would be chosen

Instead, agents send *messages*  $M$  (e.g., report their type as  $\theta'$ , or bid if we have money) to the mechanism

**Goal:** design a mechanism whose Game induces a Nash equilibrium where the outcome equals  $f(\theta)$



# A (SILLY) MECHANISM THAT DOES NOT IMPLEMENT WELFARE MAX

2 agents, 1 item

Each agent draws a private valuation for that item

Social welfare maximizing outcome: agent with greatest private valuation receives the item.

Mechanism:

- Agents send a message of  $\{1, 2, \dots, 10\}$
- Item is given to the agent who sends the lowest message; if both send the same message, agent  $i = 1$  gets the item

Equilibrium behavior: ???????????

- Always send the lowest message (1)
- Outcome: agent  $i = 1$  gets item, even if  $i = 2$  values it more



# MECHANISM DESIGN WITH MONEY

**We will assume that an agent's utility for**

- her type being  $\theta_i$ ,
- outcome  $o$  being chosen,
- and having to pay  $\pi_i$ ,

can be written as  $v_i(\theta_i, o) - \pi_i$

**Such utility functions are called **quasilinear****

- “quasi” – linear with respect to one of the raw inputs, in this case payment  $\pi_i$ , as well as a function of the rest (i.e.,  $v_i(\theta_i, o)$ )

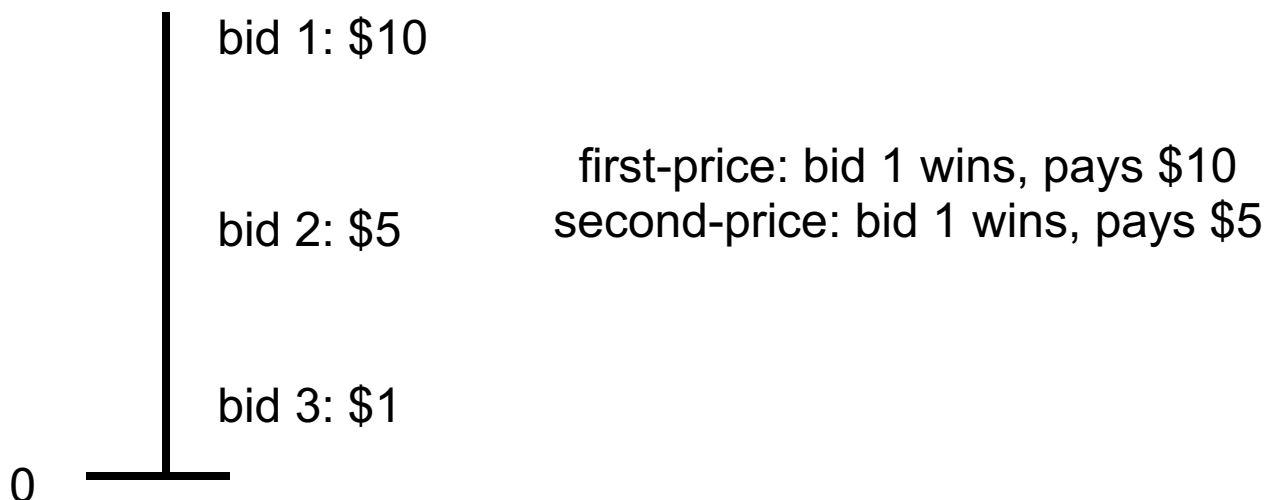
**Then, (direct) deterministic and randomized mechanisms with payments additionally specify, for each agent  $i$ , a payment function  $\pi_i : \Theta \rightarrow \mathfrak{R}$**

# EXAMPLE: (SINGLE-ITEM) AUCTIONS

**Sealed-bid** auction: every bidder submits bid in a sealed envelope

**First-price** sealed-bid auction: highest bid wins, pays amount of own bid

**Second-price** sealed-bid auction: highest bid wins, pays amount of second-highest bid



# WHICH AUCTION GENERATES MORE REVENUE?

Each bid depends on

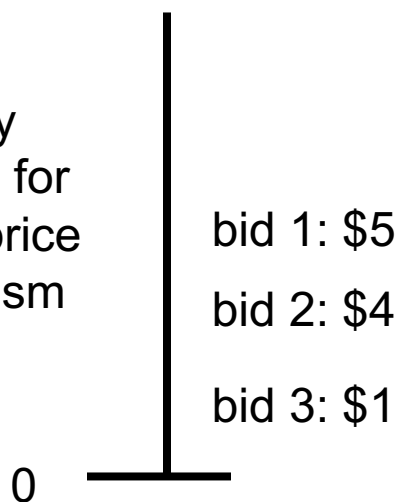
- Bidder's **true valuation** for the item (utility = valuation - payment),
- Bidder's **beliefs** over what others will bid ( $\rightarrow$  game theory),
- The **auction mechanism** used

In a first-price auction, it does not make sense to bid your true valuation  
????????????

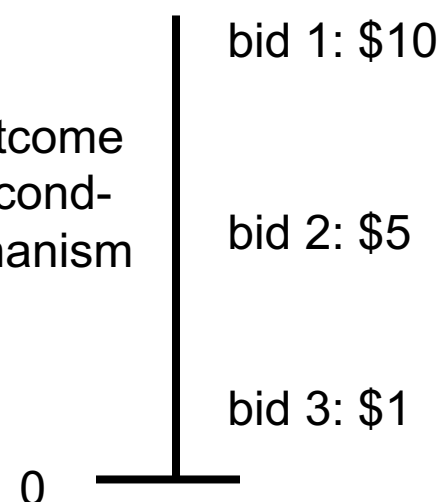
- Even if you win, your utility will be 0...

In a second-price auction, (we will see next that) it always makes sense to bid your true valuation

a likely outcome for the first-price mechanism

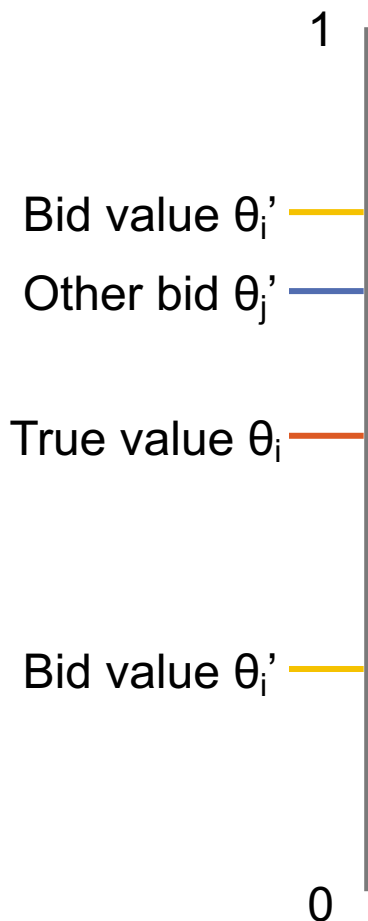


a likely outcome for the second-price mechanism



# VICKREY'S SECOND PRICE AUCTION ISN'T MANIPULABLE

(Sealed) bid on **single** item, highest bidder wins & pays second-highest bid price



Bid  $\theta_i' > \theta_i$  and win:

- Second-highest bid  $\theta_j' > \theta_i$  ?
  - Payment is  $\theta_j'$ , **pay more than valuation!**
- Second-highest bid  $\theta_j' < \theta_i$  ?
  - Payment from bidding truthfully is the same

Bid  $\theta_i' > \theta_i$  and lose: same outcome as truthful bidding

Bid  $\theta_i' < \theta_i$  and win: same outcome as truthful bidding

Bid  $\theta_i' < \theta_i$  and lose:

- Winning bid  $\theta_j' > \theta_i$  ?
  - Wouldn't have won by bidding truthfully, either
- Winning bid  $\theta_j' < \theta_i$  ?
  - Bidding truthfully would've given **positive utility**

# THE CLARKE (AKA VCG) MECHANISM

The Clarke mechanism chooses some outcome  $o$  that maximizes  $\sum_i v_i(\theta_i', o)$

To determine the payment that agent  $j$  must make:

- Pretend  $j$  does not exist, and choose  $o_{-j}$  that maximizes  $\sum_{i \neq j} v_i(\theta_i', o_{-j})$
- $j$  pays  $\sum_{i \neq j} v_i(\theta_i', o_{-j}) - \sum_{i \neq j} v_i(\theta_i', o) =$   
 $= \sum_{i \neq j} ( v_i(\theta_i', o_{-j}) - v_i(\theta_i', o) )$

We say that each agent pays the **externality** that she imposes on the other agents

- Agent  $i$ 's externality: (social welfare of others if  $i$  were absent) - (social welfare of others when  $i$  is present)

**(VCG = Vickrey, Clarke, Groves)**

# INCENTIVE COMPATIBILITY

**Incentive compatibility:** there is never an incentive to lie about one's type

A mechanism is **dominant-strategies** incentive compatible (aka **strategyproof**) if for any  $i$ , for any type vector  $\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n$ , and for any alternative type  $\theta_i'$ , we have

$$v_i(\theta_i, o(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n)) - \pi_i(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n) \geq \\ v_i(\theta_i, o(\theta_1, \theta_2, \dots, \theta_i', \dots, \theta_n)) - \pi_i(\theta_1, \theta_2, \dots, \theta_i', \dots, \theta_n)$$

A mechanism is **Bayes-Nash equilibrium (BNE)** incentive compatible if telling the truth is a BNE, that is, for any  $i$ , for any types  $\theta_i, \theta_i'$ ,

$$\sum_{\theta_{-i}} P(\theta_{-i}) [v_i(\theta_i, o(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n)) - \pi_i(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n)] \geq \\ \sum_{\theta_{-i}} P(\theta_{-i}) [v_i(\theta_i, o(\theta_1, \theta_2, \dots, \theta_i', \dots, \theta_n)) - \pi_i(\theta_1, \theta_2, \dots, \theta_i', \dots, \theta_n)]$$

# VCG IS STRATEGYPROOF

Total utility for agent  $j$  is (valuation – payment)

$$\begin{aligned} & v_j(\theta_j, o) - \sum_{i \neq j} ( v_i(\theta_i', o_{-j}) - v_i(\theta_i', o) ) \\ & = v_j(\theta_j, o) + \sum_{i \neq j} v_i(\theta_i', o) - \sum_{i \neq j} v_i(\theta_i', o_{-j}) \end{aligned}$$

But agent  $j$  cannot affect the choice of  $o_{-j}$

→  $j$  can focus on maximizing  $v_j(\theta_j, o) + \sum_{i \neq j} v_i(\theta_i', o)$

But mechanism chooses  $o$  to maximize  $\sum_i v_i(\theta_i', o)$

Hence, if  $\theta_j' = \theta_j$ ,  $j$ 's utility will be maximized!

Extension of idea: add **any** term to agent  $j$ 's payment that does not depend on  $j$ 's reported type

- This is the family of **Groves** mechanisms

# INDIVIDUAL RATIONALITY

A selfish center: “All agents must give me all their money.” – but the agents would simply not participate

- This mechanism is not **individually rational**

A mechanism is **ex-post** individually rational if for any  $i$ , for any known type vector  $\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n$ , we have

$$v_i(\theta_i, o(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n)) - \pi_i(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n) \geq 0$$

A mechanism is **ex-interim** individually rational if for any  $i$ , for any type  $\theta_i$ ,

$$\sum_{\theta_{-i}} P(\theta_{-i}) [v_i(\theta_i, o(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n)) - \pi_i(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_n)] \geq 0$$

Is the Clarke mechanism individually rational?



# WHY ONLY TRUTHFUL DIRECT-REVELATION MECHANISMS?

**Bob has an incredibly complicated mechanism in which agents do not report types, but do all sorts of other strange things**

- Bob: “In my mechanism, first agents 1 and 2 play a round of rock-paper-scissors. If agent 1 wins, she gets to choose the outcome. Otherwise, agents 2, 3 and 4 vote over the other outcomes using the STV voting rule. If there is a tie, everyone pays \$100, and ...”

**Bob: “The equilibria of my mechanism produce better results than any truthful direct revelation mechanism.”**

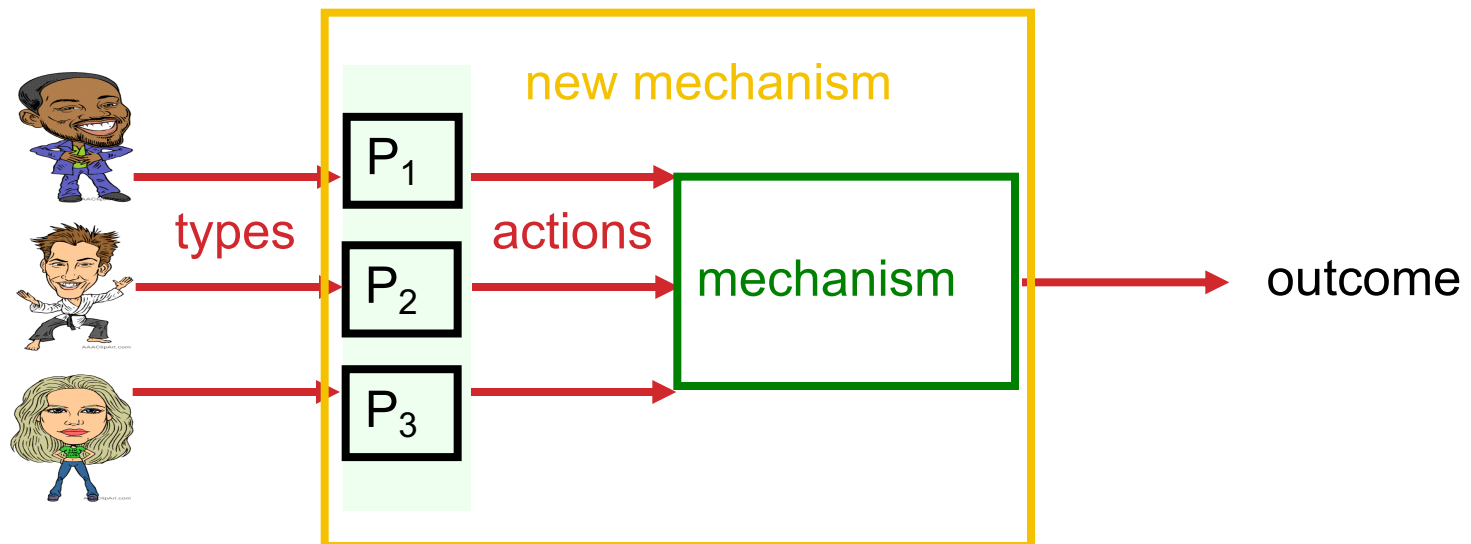
- Could Bob be right?



# THE REVELATION PRINCIPLE

For any (complex, strange) mechanism that produces certain outcomes under strategic behavior (dominant strategies, BNE)...

... there exists a {**dominant-strategies, BNE**} **incentive compatible direct-revelation** mechanism that produces the same outcomes!



# REVELATION PRINCIPLE IN PRACTICE

## “Only direct mechanisms needed”

- But: strategy formulator might be complex
  - Complex to determine and/or execute best-response strategy
  - Computational burden is pushed on the center (i.e., assumed away)
  - Thus the revelation principle might not hold in practice if these computational problems are hard
  - This problem traditionally ignored in game theory
- But: even if the indirect mechanism has a unique equilibrium, the direct mechanism can have additional bad equilibria

# REVELATION PRINCIPLE AS AN ANALYSIS TOOL

**Best direct mechanism gives tight upper bound on how well any indirect mechanism can do**

- Space of direct mechanisms is smaller than that of indirect ones
- One can analyze all direct mechanisms & pick best one
- Thus one can know when one has designed an optimal indirect mechanism (when it is as good as the best direct one)

# COMPUTATIONAL ISSUES IN MECHANISM DESIGN

## Algorithmic mechanism design

- Sometimes standard mechanisms are too hard to execute computationally (e.g., Clarke requires computing optimal outcome)
- Try to find mechanisms that are easy to execute computationally (and nice in other ways), together with algorithms for executing them

## Automated mechanism design

- Given the specific setting (agents, outcomes, types, priors over types, ...) and the objective, have a **computer** solve for the best mechanism for this particular setting

**When agents have computational limitations, they will not necessarily play in a game-theoretically optimal way**

- Revelation principle can collapse; need to look at nontruthful mechanisms

**Many other things (computing the outcomes in a distributed manner; what if the agents come in over time (online setting); ...) – many good project ideas here ☺.**

# **RUNNING EXAMPLE: MECHANISM DESIGN FOR KIDNEY EXCHANGE**

# THE PLAYERS AND THEIR INCENTIVES

## **Clearinghouse cares about global welfare:**

- How many patients received kidneys (over time)?

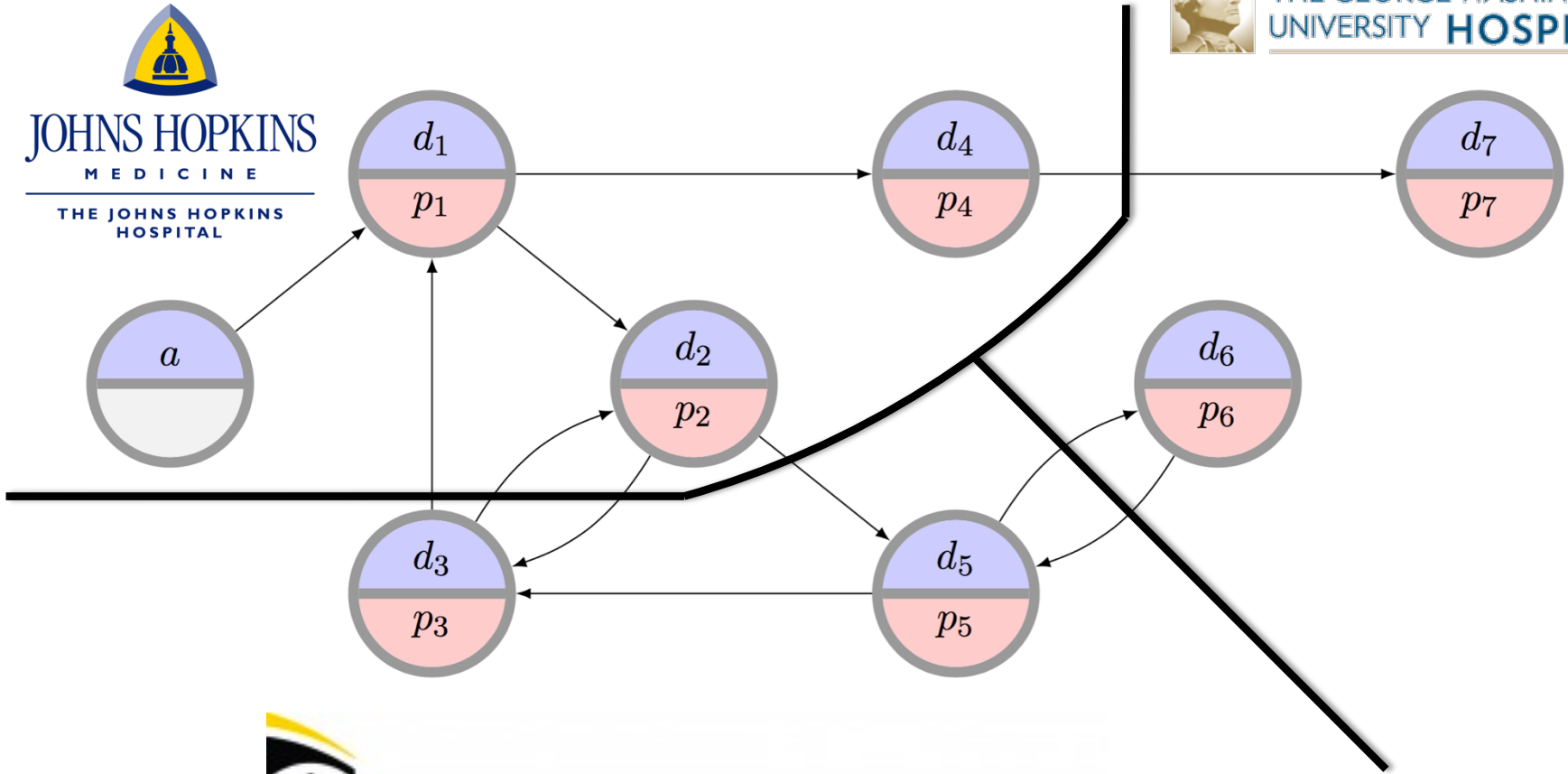
## **Transplant centers care about their individual welfare:**

- How many of my own patients received kidneys?

## **Patient-donor pairs care about their individual welfare:**

- Did I receive a kidney?
- (Most work considers just clearinghouse and centers)

# PRIVATE VS GLOBAL MATCHING





# MODELING THE PROBLEM

**What is the type of an agent?**

**What is the utility function for an agent?**

**What would it mean for a mechanism to be:**

- **Strategyproof**
- **Individually rational**
- **Efficient**

# KNOWN RESULTS

**Theory** [Roth&Ashlagi 14, Ashlagi et al. 15, Toulis&Parkes 15]:

- **Can't have a strategy-proof and efficient mechanism**
- **Can get “close” by relaxing some efficiency requirements**
- **Even for the **undirected** (2-cycle) case:**
  - No deterministic SP mechanism can give 2-eps approximation to social welfare maximization
  - No randomized SP mechanism can give 6/5-eps approx
- **But! Ongoing work by a few groups hints at **dynamic models** being both more realistic and less “impossible”!**

**Reality: transplant centers strategize like crazy!** [Stewart et al. 13]



# **NEXT CLASS/LATER THIS CLASS: COMBINATORIAL OPTIMIZATION**



**ALSO: PROJECTS!**