

APPLIED MECHANISM DESIGN FOR SOCIAL GOOD

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Lecture #17 – 04/07/2020

CMSC828M
Tuesdays & Thursdays
2:00pm – 3:15pm

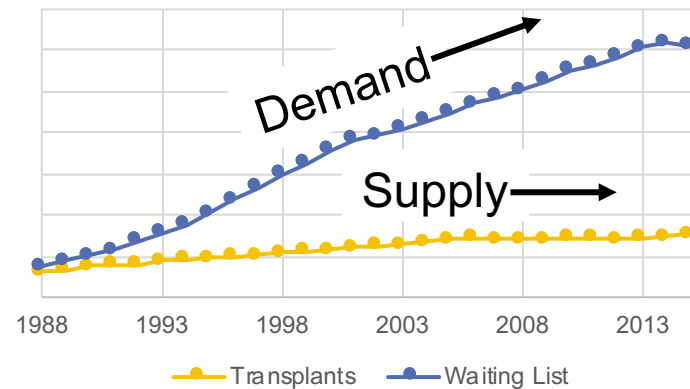


COMPUTER SCIENCE
UNIVERSITY OF MARYLAND

**THIS CLASS:
ORGAN EXCHANGE**

KIDNEY TRANSPLANTATION

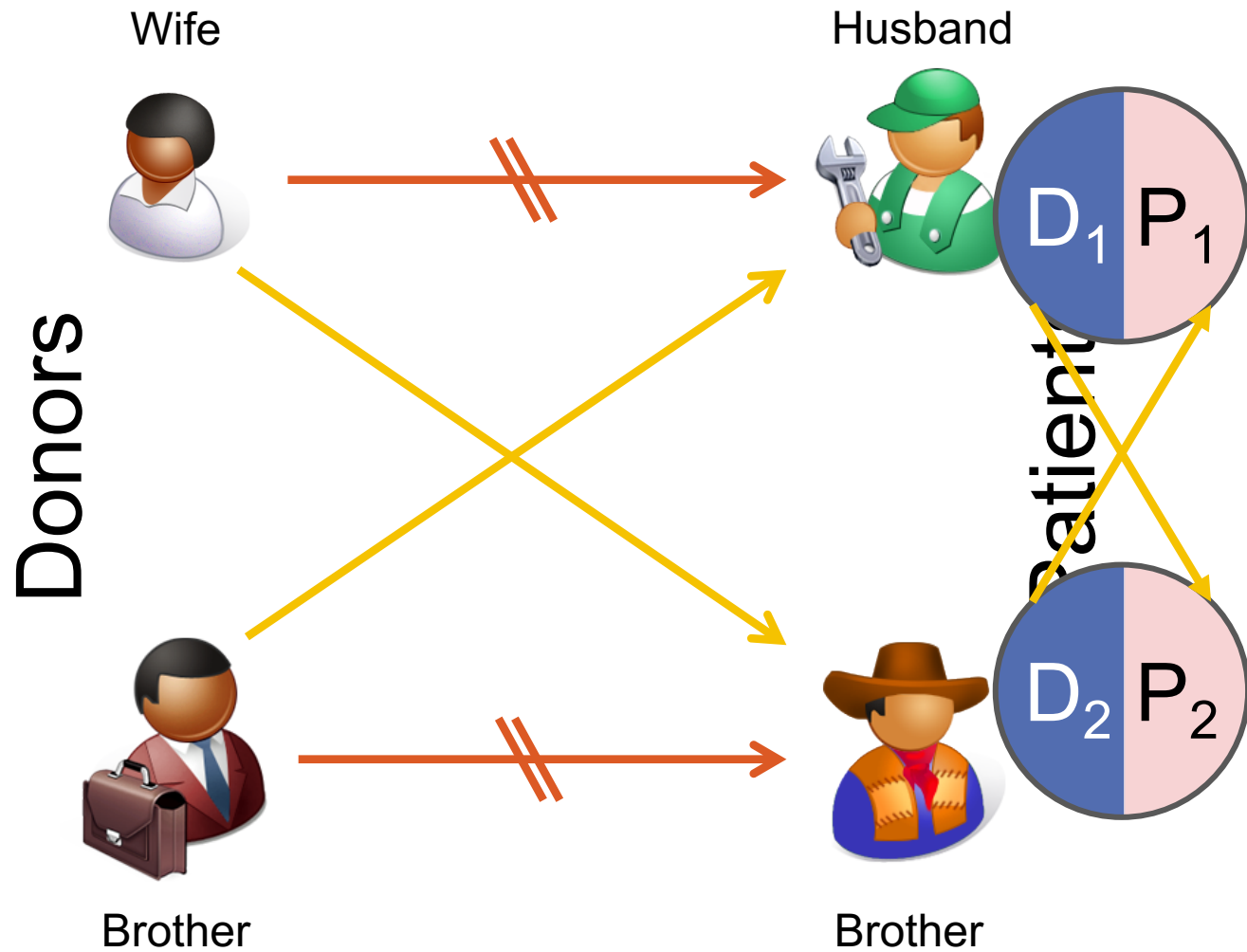
- **US waitlist: over 100,000**
 - Over 35,000 added per year
- **~4500 people died while waiting**
- **~12000 people received a kidney from the deceased donor waitlist**



- (See last class' lecture on deceased donor allocation.)
- **~6000 people received a kidney from a living donor**
 - Some through **kidney exchange**

*Last time,
I promise!*

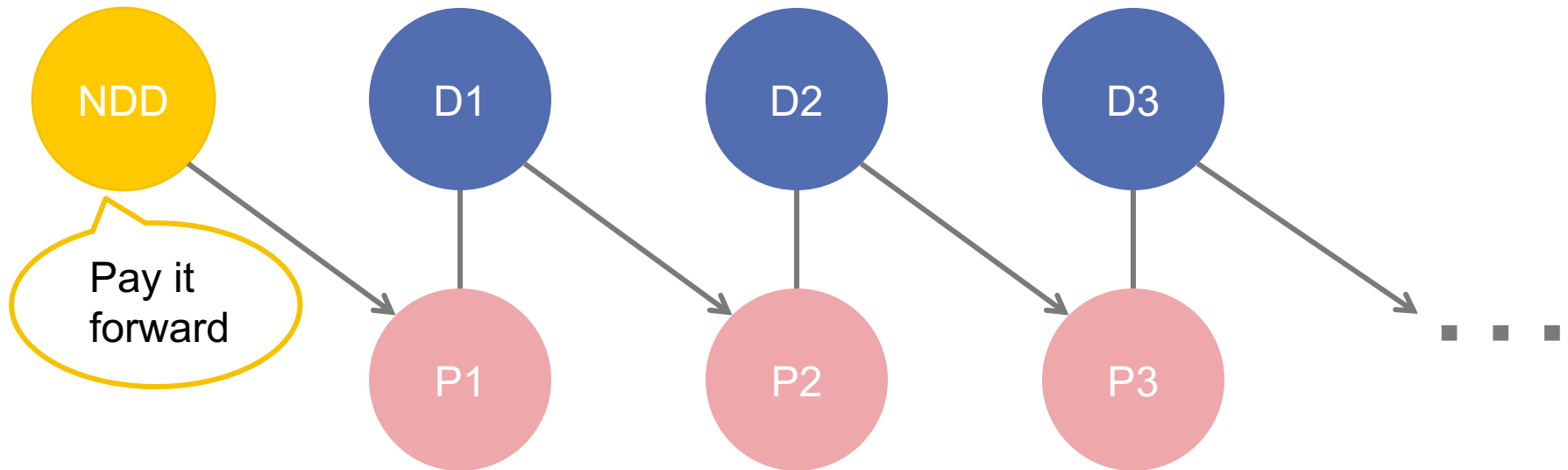
KIDNEY EXCHANGE



(2- and 3-cycles, all surgeries performed simultaneously)

NON-DIRECTED DONORS & CHAINS

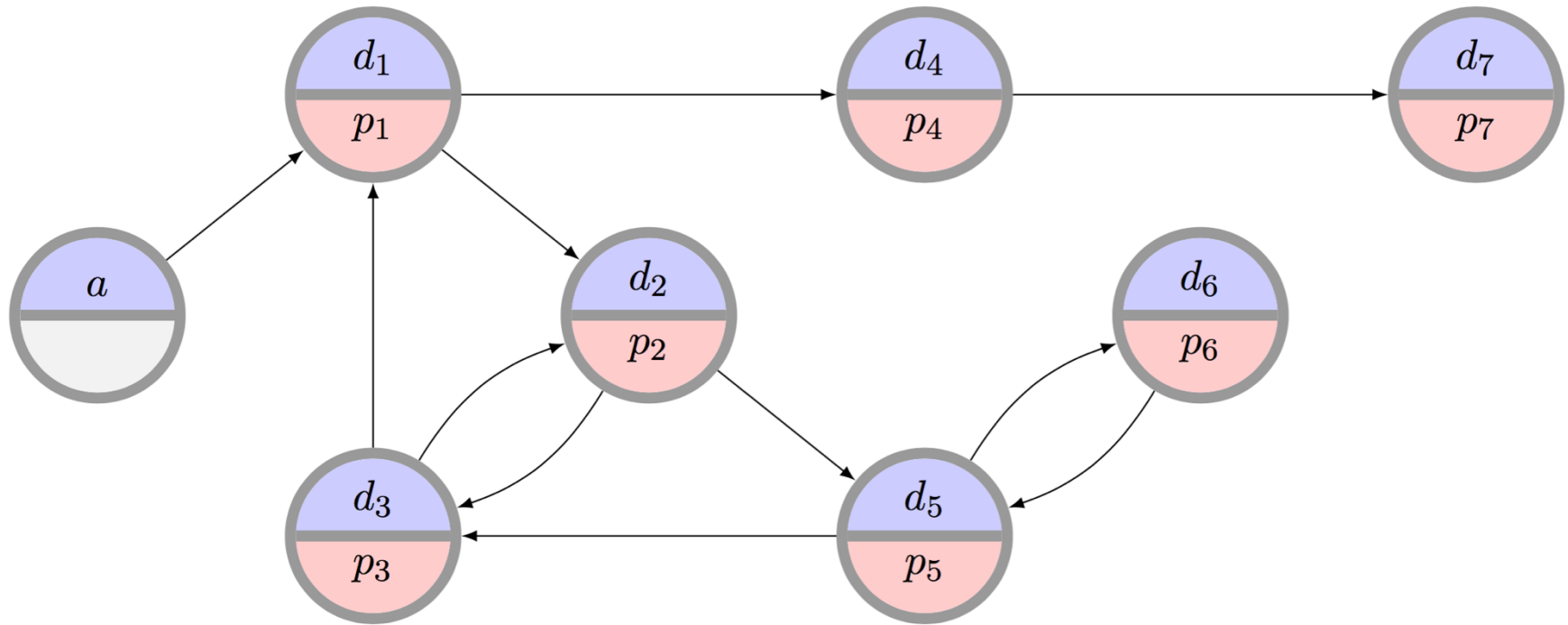
[Rees et al. 2009]



Not executed simultaneously, so no length cap required based on logistic concerns ...

... but in practice edges fail, so often some finite cap is used!

THE CLEARING PROBLEM



The **clearing problem** is to find the “best” disjoint set of cycles of length at most L , and chains (maybe with a cap K)

- Very hard combinatorial optimization problem that we will focus on in the succeeding two lectures.

MANAGING INCENTIVES

Clearinghouse cares about global welfare:

- How many patients received kidneys (over time)?

Transplant centers care about their individual welfare:

- How many of my own patients received kidneys?

Patient-donor pairs care about their individual welfare:

- Did I receive a kidney?
- (Most work considers just clearinghouse and centers)

INDIVIDUAL RATIONALITY (IR)

Will I be better off participating in the mechanism than I would be otherwise?

Long-term IR:

- In the long run, a center will receive at least the same number of matches by participating

Short-term IR:

- At each time period, a center receives at least the same number of matches by participating

STRATEGY PROOFNESS

Do I have any reason to lie to the mechanism?

In any state of the world ...

- { time period, past performance, competitors' strategies, current private type, etc }

... a center is not worse off reporting its full private set of pairs and altruists than reporting any other subset

→ No reason to strategize

EFFICIENCY

Does the mechanism result in the absolute best possible solution?

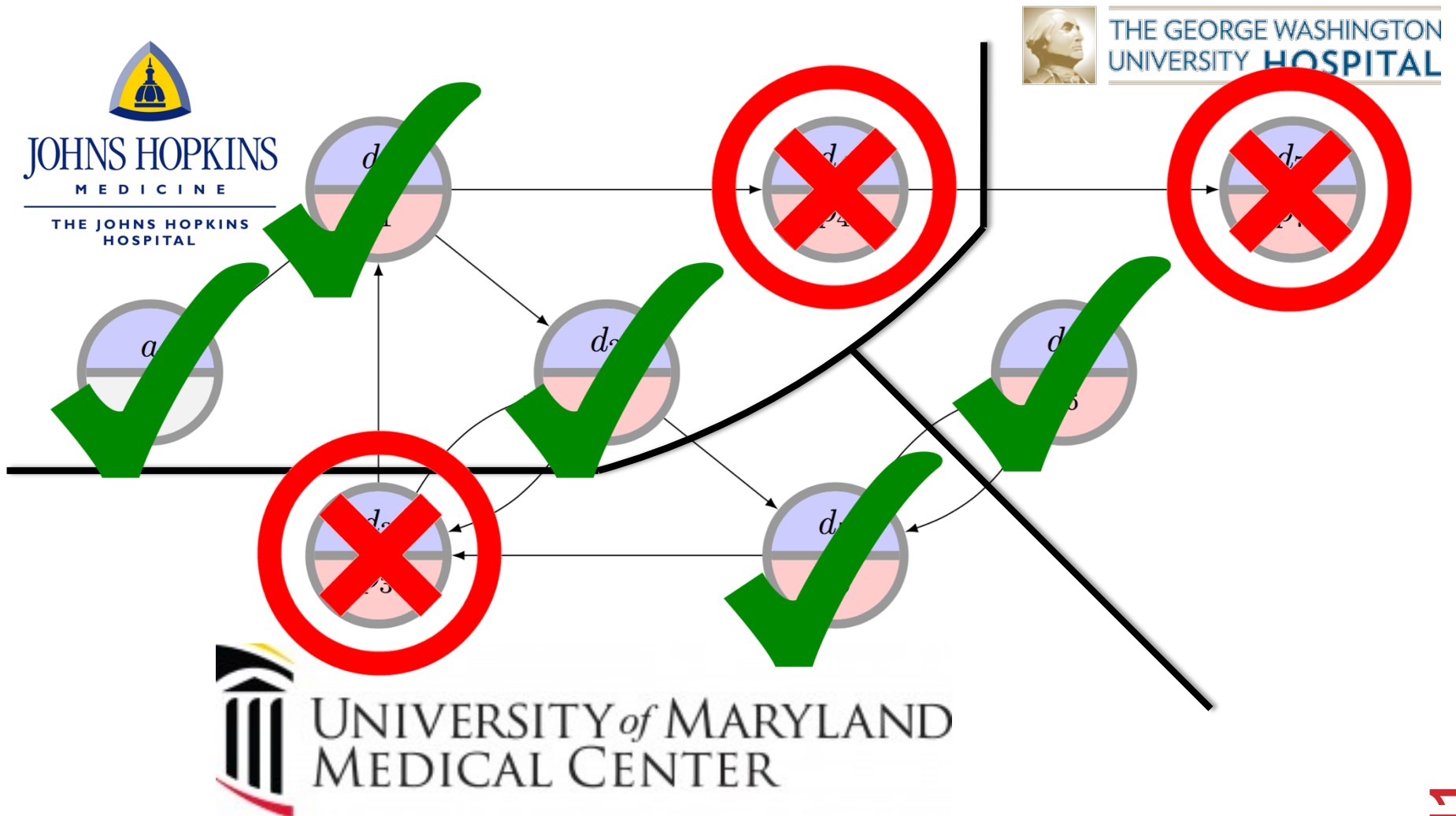
Efficiency:

- Produces a maximum (i.e., max global social welfare) matching given all pairs, regardless of revelation

IR-Efficiency:

- Produces a maximum matching constrained by short-term individual rationality

PRIVATE VS GLOBAL MATCHING



0%

FIRST: ONLY CYCLES (NO CHAINS)

THE BASIC KIDNEY EXCHANGE GAME

[Ashlagi & Roth 2014, and earlier]

Set of n transplant centers $T_n = \{t_1 \dots t_n\}$, each with a set of incompatible pairs V_h

Union of these individual sets is V , which induces the underlying compatibility graph

Want: all centers to participate, submit full set of pairs

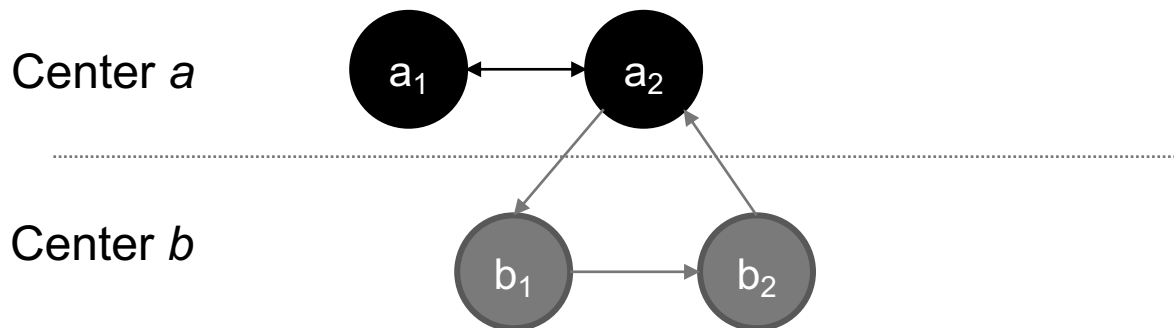
An allocation M is **k -maximal** if there is no allocation M' that matches all the vertices in M and also more

- Note: k -efficient \rightarrow k -maximal, but not vice versa

INDIVIDUALLY RATIONAL?

[Ashlagi & Roth 2014, and earlier]

- Vertices a_1, a_2 belong to center a ,
 b_1, b_2 belong to center b
- Center a could match 2 internally ????????????????????
- By participating, matches only 1 of its own
- Entire exchange matches 3 (otherwise only 2)

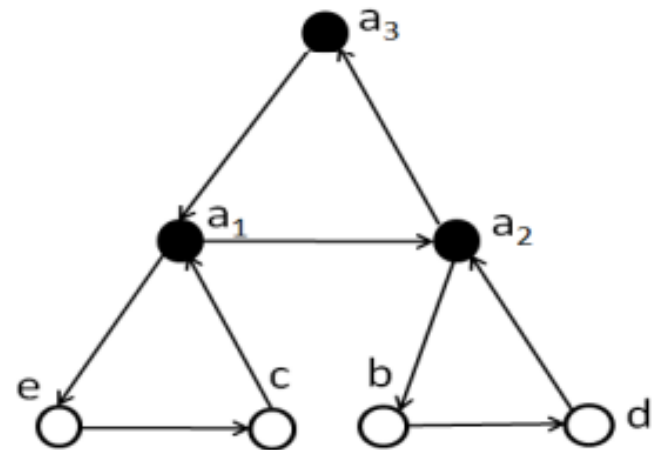


IT CAN GET MUCH WORSE

[Ashlagi & Roth 2014, and earlier]

Theorem: For $k > 2$, there exists G s.t. no IR k -maximal mechanism matches more than $1/(k-1)$ -fraction of those matched by k -efficient allocation

- **Bound is tight**
- **All but one of a 's vertices is part of another length k exchange (from different agents)**
- **k -maximal and IR if a matches his k vertices (but then nobody else matches, so k total)**
- **k -efficient to match $(k-1)*k$**



Example: $k=3$

RESTRICTION #1 [Ashlagi & Roth 2014, and earlier]

Theorem: For all k and all compatibility graphs, there exists an IR k -maximal allocation

Proof sketch: construct k -efficient allocation for each specific hospital's pool V_h

Repeatedly search for larger cardinality matching in an entire pool that keeps all already-matched vertices matched (using augmenting matching algorithm from Edmonds)

Once exhausted, done

RESTRICTION #2 [Ashlagi & Roth 2014, and earlier]

Theorem: For $k=2$, there exists an IR 2-efficient allocation in every compatibility graph

Idea: Every 2-maximal allocation is also 2-efficient

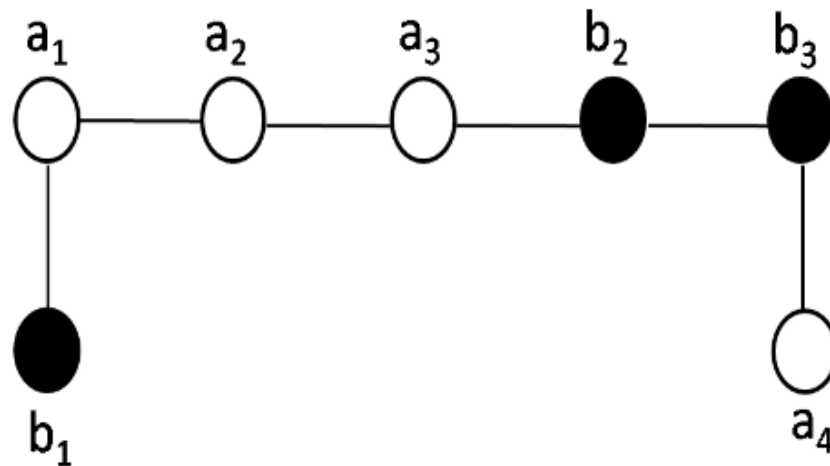
- This is a PTIME problem with, e.g., a standard $O(|V|^3)$ bipartite augmenting paths matching algorithm

By Restriction #1, 2-maximal IR always exists \rightarrow this 2-efficient IR always exists

RESTRICTION #3 [Ashlagi et al. 2015]

Theorem: No IR mechanism is both maximal and strategyproof (even for $k=2$)

Suppose mechanism is IR and maximal . . .



MORE NEGATIVE MECHANISM DESIGN RESULTS

[Ashlagi et al. 2015]

Just showed IR + strategyproof \rightarrow not maximal

No IR + strategyproof mechanism can guarantee more than $\frac{1}{2}$ -fraction of efficient allocation

- Idea: same counterexample, note either the # matched for hospital a ≤ 3 , or # matched for hospital b ≤ 2 . Proof by cases follows

No IR + strategyproof randomized mechanism can guarantee $\frac{7}{8}$ -fraction of efficiency

- Idea: same counterexample, bounds on the expected size of matchings for hospitals a, b

HOPELESS ...?



DYNAMIC, CREDIT-BASED MECHANISM [Hajaj et al. AAAI-2015]

Repeated game

Centers are risk neutral, self interested

Transplant centers have (private) sets of pairs:

- Maximum capacity of $2k_j$
- General arrival distribution, mean rate is k_j
- Exist for one time period

Centers reveal subset of their pairs at each time period, can match others internally

CREDITS

Clearinghouse maintains a credit balance c_i for each transplant center over time

High level idea:

- **REDUCE** c_i : center i reveals fewer than expected
- **INCREASE** c_i : center i reveals more than expected

- **REDUCE** c_i : mechanism tiebreaks in center i 's favor
- **INCREASE** c_i : mechanism tiebreaks against center i

Also remove centers who misbehave "too much."

Credits now → matches in the future

THE DYNAMIC MECHANISM

1. Initial credit update

- Centers reveal pairs
- Mechanism updates credits according to k_i

2. Compute maximum global matching

- Gives the utility U_g of a max matching

3. Selection of a final matching

- Constrained to those matchings of utility U_g
- Take c_i into account to (dis)favor utility given by matching to a specific center i
- Update c_i based on this round's (dis)favoring

4. Removal phase if center is negative for “too long”

THEORETICAL GUARANTEES

Theorem: No mechanism that supports cycles and chains can be both long-term IR and efficient

Theorem: Under reasonable assumptions, the prior mechanism is both long-term IR and efficient

LOTS OF OPEN PROBLEMS HERE

Dynamic mechanisms are more realistic, but ...

- Vertices disappear after one time period
- All hospitals the same size
- No weights on edges
- No uncertainty on edges or vertices
- Upper bound on number of vertices per hospital
- Distribution might change over time
- ...

Project?

WHAT DO EFFICIENT MATCHINGS EVEN LOOK LIKE ...?

Next class: given a specific graph, what is the “optimal matching”

This class: given a **family of graphs, what do ”optimal matchings” tend to look like?**

Use a stylized random graph model, like [Saidman et al. 2006]:

- Patient and donor are drawn with blood types randomly selected from PDF of blood types (roughly mimics US makeup), randomized “high” or “low” CPRA
- Edge exists between pairs if candidate and donor are ABO-compatible and tissue type compatible (random roll weighted by CPRA)

RANDOM GRAPH PRIMER

Canonical Erdős-Rényi random graph $G(m,p)$ has m vertices and an (undirected) edge between two vertices with probability p

- Let Q be the property of “there exists a perfect matching” in this graph

The convergence rate to 1 (i.e., “there is almost certainly a near perfect matching in this graph) is exponential in p

- $\Pr(G(m,m,p) \text{ satisfies } Q) = 1 - o(2^{-mp})$
- At least as strong with non-bipartite random graphs

Early random graph results in kidney exchange are for “in the large” random graphs that (allegedly) mimic the real compatibility graphs

- All models are wrong, but some are useful?

A STYLIZED ERDŐS-RÉNYI-STYLE MODEL OF KIDNEY EXCHANGE

In these random (ABO- & PRA-) graphs:

- # of O-{A, B, AB} pairs > {A, B, AB}-O pairs
- # of {A, B}-AB pairs > AB-{A, B} pairs
- Constant difference between # A-B and # B-A

Idea #1: O-candidates are hard to self-match

Idea #2: {A, B}-candidates are hard to self-match

Idea #3: “symmetry” between A-B and B-A (equally hard to self-match, give or take)

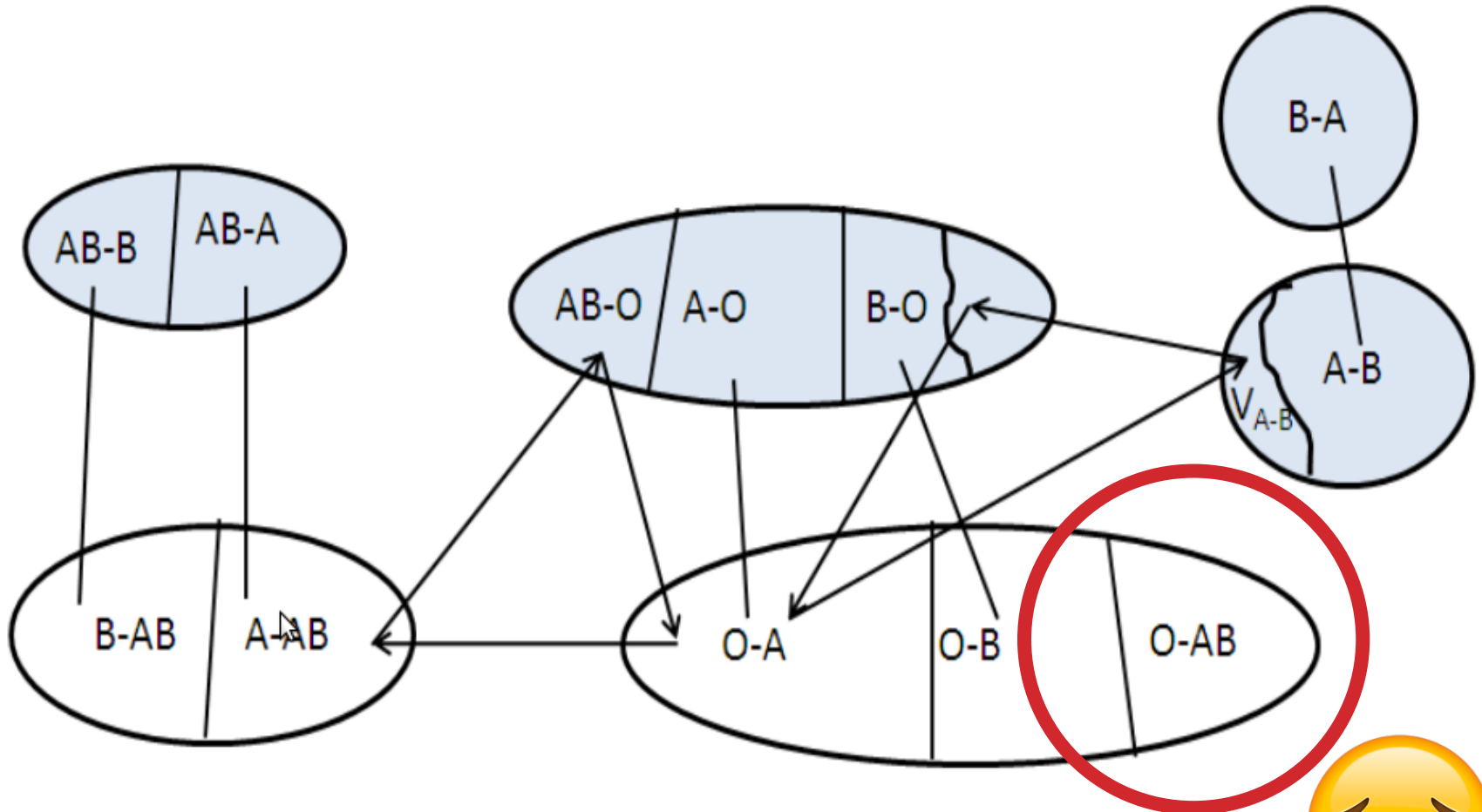
EFFICIENT MATCHING IN DENSE GRAPHS WITH ONLY CYCLES

Under some other assumptions about PRA ...

Almost every large random (ABO- & PRA-) graph has an efficient allocation that requires exchanges of size at most 3 with the following:

- X-X pairs are matched in 2- or 3-way exchanges with other X-X pairs (so-called “self-demand”)
- B-A pairs are 2-matched with A-B pairs
- The leftovers of {A-B or B-A} are 3-matched with “good” {O-A, O-B} pairs and {O-B, O-A pairs}
- 3-matches with {AB-O, O-A, A-AB}
- All the remaining 2-matched as {O-X, X-O}

VISUALLY ...



**NEXT CLASS:
OPTIMAL BATCH CLEARING OF ORGAN
EXCHANGES**